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TREATISE ON THE TIDES

JAMES PEARSON, M.A., F.R.A.S.

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AN ELEMENTARY TREATISE
ON
THE TIDES,

BASED UPON THAT OF THE LATE

SIR J. W. LUBBOCK, BART., F.R.S.,

TO WHICH IS ADDED

A NEWLY-DEVISED METHOD OF COMPUTATION OF THE HEIGHTS OF HIGH WATER AT LIVERPOOL, WITH FACTORS FOR OTHER PORTS AND TABLES ADOPTED BY THE ADMIRALTY,

BY

JAMES PEARSON, M.A.,F.R.A.S.,

Late Scholar of Trinity College, Cambridge, and Vicar of Fleetwood.

"HE GATHERETH THE WATERS OF THE SEA TOGETHER AS IT WERE UPON A HEAP, AND LAYETH UP THE DEEP AS IN A TREASURE HOUSE."—*Psalm* 33—7.

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PREFACE.

DURING the last ten years I have been making a continuous series of observations on the tides, and have compared them with the results of theory. It is a subject which becomes more fascinating, the more fully we seek acquaintance with it. Few care to give to it the attention it deserves, and fewer still have the opportunity of noting the effect of barometric changes and atmospheric gradients, so as to eliminate these disturbing causes from the general effects. Mrs. Somerville designated the state of the theory of the tides in her time as a "reproach to science," and even Dr. Whewell, in his History of the Inductive Sciences, complains of want of success. Under these circumstances, there is every encouragement to persevere in the study. Like an unploughed field, it demands cultivation, and the more so, because so few care to work in it. The method of computation which my own investigation has led me to devise, has been proved to be true to nature. Having been adopted by the Admiralty Authorities for several years past, the tables founded upon it will be printed in next year's volume of the Admiralty Tables, and I have therefore thought this a fitting opportunity of making the details of the calculation more generally known and understood. The treatise of Sir John Lubbock, on the principles of which the present work is based, has long been out of print, and the literature of the subject is contained in isolated tracts, pamphlets, and reports, which are very inaccessible. It is hoped, therefore, that this small volume may supply a want, and be favorably received by those who take an interest in the progress of science.

J. P.

The Vicarage, Fleetwood,
October 1st, 1881.

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THE TIDES.

CHAPTER I.

FACTS AND HISTORY.

THE ocean is often agitated by winds and tempests which disturb its equilibrium; but these causes are inadequate to produce the phenomena of its motion, for in the calmest weather it rises and falls alternately. It is *high water* at a given place a certain time after the moon has passed the meridian of that place, after which it falls until it is *low water*, when it again gradually rises. These phenomena recur after nearly the same intervals of time, and are called the *tides*.

The interval of time between low water and the following high water is called *flood tide*, and that between high water and the following low water is called *ebb tide*. If the interval of time between two successive passages of the moon over the same meridian be called a *lunar day*, there are generally *two* high tides in a lunar day.

The time of the moon's synodic revolution, or period of her mean conjunctions, is 29·53059 days. Hence the mean interval between successive high tides is 12h. 24m. 22s. This circumstance furnishes an explanation of the fact, that on two days in each month, there is only *one* tide in the day, and not two. For suppose high water to occur at 11h. 45m. p.m. on any day, as Monday, *i.e.*, shortly before midnight, according to the civil reckoning of time; then, agreeably with what has been stated, the succeeding tide will be on Tuesday at 0h. 10m. p.m. Hence there will be no tide on Tuesday morning; *i.e.*, between midnight and noon of that day.

The interval, however, which elapses between the moon's transit and the time of high water is subject to irregularities, which depend principally on her angular distance from the sun. These variations depend also upon the distances of both luminaries from the earth, and upon their declinations. The heights of high water also vary:

they are greatest soon after the moon is in conjunction, and least soon after she is in quadrature; the one are called *spring* the other *neap* tides. These heights depend also on the declinations and distances of the luminaries.

Then, again, the times occupied by the tides in flowing and ebbing are unequal, with few exceptions. Thus, at Liverpool, the interval from low water to high water is about 5¼ hours, whilst the interval from high water to low water is 6¾ hours, on the average. Moreover the heights of spring tides vary considerably at different seasons of the year. They are at a maximum, *i.e.* the tides are highest, at the equinoxes, in March and September, when the sun is near the equator, and the moon crosses the equator at full and change. They are at a minimum, *i e.* the tides are lowest, in June and December, when the sun is furthest from the equator, and the moon at full and change, attains the same position. In the case of *equinoctial* tides, as the former are called, the joint action of the sun and moon is exerted along the greatest possible circumference of the globe, and therefore over the largest extent of waters; whilst in the latter case, that of the solstitial tides, the joint action takes place along parallels of latitude of smaller circumference. It may also be remarked that the higher the tide rises, the lower is its ebb, which is easily explained, because the tide not being at its height in different places at the same instant, the extra drain of the waters in one locality contributes to swell the redundance of waters in another. Where the successive tides of the same day are very unequal, the ebb which follows the greater of these tides is lower than that which follows the less.

The evident connection between the period of the tides, and the phases of the moon, led philosophers to attribute these phenomena to lunar influences long before the true theory was understood. The rise and fall of the tides of the ocean presented a spectacle too remarkable and too important to the social and commercial interests of mankind not to have attracted notice at an early period in the progress of knowledge; but the actual records which we find in ancient authors concerning the tides are exceedingly few and scanty. The reason of this seems to be that the principal seats of civilization and learning were situated in regions remote from those great oceans and seas, where these phenomena are most developed. Herodotus, book ii, chap. 11, speaking of a long but narrow bay diverging from the Red Sea, in his description of the land of Egypt, B.C. 445, says—"In this bay the tide daily ebbs and flows." This is probably the first historical observation which has come down to our times. Pliny, book ii, chap. 97, as quoted by Lalande in the fourth volume of his Astronomy, p. 8, has the following very remarkable passage on the matter—"That the tides of the ocean ebb and flow is exceedingly wonderful: these are produced under various circumstances, but the cause is in the sun and moon," and he proceeds to describe their course and order very accurately. He died in A.D. 79. In Lingard's Antiquities of the Anglo-Saxon Church, chap. 10, we have a succinct account of the knowledge of the subject possessed by the Saxons. "From their insular situation," he observes, "the Saxons could not be ignorant of the interesting phenomena of the tides, and Bede seems to have suspected the existence of that cause, the discovery of which has immortalized the name of Newton." Bede lived A.D. 700, and he connected the ebb and flow with the action of the moon, with whose motion they so accurately corresponded, that he was tempted to believe that the waters were drawn towards that luminary by some invisible influence, and after a certain time were permitted to return to their former situation. In his work *de ratione temporum*, c. 27, p. 116, he says—"the tide is drawn forward as if by certain influences of the moon, and again it is poured back to its natural bounds, when the force alluded to ceases." He does not, however, venture to speculate on the nature of this attraction, but confines himself to the following enumeration of the particulars in which the motions of the moon and of the ocean appear to coincide. "As the moon daily recedes twelve degrees from the sun, so, on an average, the tides are daily retarded four points" (eight and forty minutes) "in their approach to the shore. Some days before the conjunction they seem to increase: and from the fifth to the twelfth, from the twentieth to the twenty-seventh day, they continually diminish. But the gradations of increase and decrease are not perfectly regular, and these anomalies may be ascribed, perhaps to the impulse or resistance of the winds, more probably to the agency of some unknown power." The Anglo-Saxon, however, was able to correct an erroneous opinion of former philosophers. It had been pretended that in every part of the ocean the waters began to rise at the same moment; but daily observation authorised him to assert that, "on the eastern coast of Britain, the tide was propagated from the north to the south, and that it reached the mouth of the river Tyne before it washed the coast of the Deiri."

Descartes, after Seleucus, as quoted by Plutarch, imagined that the moon pressing on the surface of the waters immediately beneath it, forced those waters to rise along the distant coasts, attaining their maximum of elevation 90 degrees from the spot at which the moon was in the zenith.

The gravitation of the waters of the ocean towards the moon was clearly understood by Kepler, A.D. 1605, although he failed to indicate the law of gravitation. In one of his Dialogues he writes—"the sphere of attractive virtue which is in the moon extends as far as the earth and entices up the waters, but as the moon flies rapidly across the zenith, and the waters cannot follow so quickly, a flow of the waters is occasioned in the torrid zone towards the westward." And again in his *New Philosophy*—"There are two primary causes of the motion of the seas—the moon and the diurnal revolution. The moon does not act on the seas by its rays or its light. How then? Certainly by the common effort of the bodies, and (to explain it by something similar) by their magnetic attraction. It should be known, in the first place, that the whole quantity of water is not contained in the sea and rivers, but that the mass of the earth (I mean this globe) contains moisture and spirit much deeper even than the sea. The moon draws this out by sympathy, so that they burst forth on the arrival of the moon, in consequence of the attraction of that luminary, &c."

Galileo expresses his regret that so ingenious a philosopher as Kepler should have given such an explanation as that of which a portion has just been quoted. His fourth dialogue is a development of a treatise on this and other subjects which was sent to the Archduke Leopold in 1618. His theory was erroneous, but it required a further advance in the science of motion to point out its insufficiency. He attributed the tides to the rotation of the earth combined with its revolution round the sun. From the earliest ages the problem had been regarded as most difficult, insomuch that it had been proverbially designated "the grave of human curiosity:" so Galileo, having a theory of his own, clung to it with extraordinary tenacity.

Wallis, in 1666, in his letters to Mr. Boyle, derives the phenomena of the tides from the consideration that it is the common centre of gravity of the moon and earth which describes an orbit round the sun, while they revolve about this common centre. He, however, appears to have conjectured the principle of gravitation, though it is scarcely clear that he understood how the tides were caused by its action. These letters are published in his works, and in the Philosophical Transactions, vol. i, p. 263. In answer to an objection which was made to him in the form of a query, "How two bodies which have no tie can have a common centre of gravity?" Wallis says, "It is harder to show how they have, than that they have it. That the loadstone and iron have somewhat equivalent to a tie, though we see it not, yet by the effects we know. And it would be easy to show that two loadstones at once applied in different positions to the same needle at a convenient distance, will draw it, not to a point directly to either of them, but to some point between both. . . . . As to the present case, how the earth and moon are connected, I will not undertake to show, nor is it necessary to my purpose; but that there is somewhat that does connect them, *as much as what connects the loadstone and iron which it draws*, is past doubt to those who allow them to be carried about the sun as one aggregate or body, whose parts keep a respective position to one another, like as Jupiter with his four satellites and Saturn with his one. Some tie there is which makes those satellites attend their lords and move in a body, though we do not see that tie, nor hear the words of command." Bacon's treatise "on the flow and ebb of the sea" does not contain an accurate solution of the question; he conceives the phenomena to have been attributed to the influence of the moon "by common but unsupported conjecture." Bacon's treatise appeared before A.D. 1619.

In the Philosophical Transactions, vol. 13, p. 10, we have a correct tide table showing the true times of high water at London Bridge for every day in the year 1683. This was due to Flamstead, who was appointed first Astronomer-Royal at a salary of £100 a year. His attention had been turned to the subject from his having frequently occasion to go to Greenwich by water.

To Sir Isaac Newton, however, belongs the glory of discovering and expounding the true theory of the tides. Before the epoch of Newton all explanations were at best but vague surmises. His theory explained, with singular felicity, all the prominent circumstances attending them—the difference of spring and neap tides, with the effect of the moon's and sun's declination and parallax. It was about this time, also, that the Royal Society of England and the Academy of France made attempts to collect practical observations, and in 1740 prizes were offered for the best treatise on the whole subject. The treatises selected were those contributed by Euler, D'Alembert, and Bernoulli, and that of Bernoulli contained a table for computing the times of high water, which with proper modifications can still be used for the port of London. Bernoulli's theory amounts to supposing that, if the earth was a perfect sphere or spheroid, covered throughout by an ocean, the ocean would assume the same form, at any given instant, which it would take if the forces then acting upon it were invariable in magnitude and direction. In other words Bernoulli supposes the form of the ocean encompassing a perfect sphere or spheroid to be that which it would assume at any moment, if it, and the luminaries which produce the tides, were at rest. If we suppose this state of things to obtain approximately in the great expanse of water in the southern hemisphere, and that the tide is regularly propagated thence to our coast with a velocity independent of the position of the sun and moon, it is remarkable how closely, in many circumstances, the actual results agree with the times and heights predicted. The phenomena of the tides, however, are composite, and the laws which would obtain in the perfect sphere are liable to many modifications in the progress of the tide-wave, so that it is difficult in various instances to determine whether the errors arise from any of the small corrections which would be required in the perfect sphere or to the different velocities with which spring and neap tides travel in narrow channels.

In 1774 Laplace took up the subject. Availing himself of the discoveries of D'Alembert in the Theory of Hydrodynamics, he attempted to deduce, from the equations of motion of fluids, the principal phenomena of the tides. The integration, however, of these equations presents great difficulties, even in the case in which the depth of the ocean is constant, and the solid nucleus but little different from a sphere.

Finally, Laplace had recourse to the following indirect consideration, viz.: "*that the state of any system, in which the primitive conditions have disappeared through the resistances which its motion encounters, is periodical with the forces which act upon it.*" Hence he concludes that if the system is disturbed by a periodic force, expressed by a series of cosines of variable angles, the height of the tide is represented by a similar series, in which the *periods* of the *arguments* are the same, but the *epochs* and *coefficients* are different. This amounts to expressing the height of the tide (or depth of the ocean) by an expression in which everything is arbitrary, except the *periods* of the arguments. Sometimes Laplace appears to treat the phenomena at Brest as derived, but in other places he appears to consider that they must coincide with those which would obtain at that geographical latitude in the perfect sphere.

The attention of Laplace does not appear to have been directed to the construction of tide tables for predicting the time and height of high water at any port.

The labours of Sir John Lubbock and Dr. Whewell should next be referred to. By Dr. Whewell an attempt was made to trace the progress of the tide into all the seas of the globe by drawing upon maps of the ocean what he calls "cotidal-lines," *i.e.* lines marking the contemporaneous position of the summits of the great wave which carries high water from shore to shore. These interesting maps exhibit (1) the general courses of the tide-waves over the whole globe, and (2) the circulation of the subordinate or derived waves around the seas of Great Britain and Ireland. By Sir John Lubbock careful comparison was made between theory and observation, and tables were obtained for the effect of the moon's declination, parallax, and transit. These tables represented the facts with a remarkable degree of precision ; but the acrimony with which the errors were pointed out and dwelt upon showed the jealousy of certain practical tide table calculators, who, having obtained an insight into Bernoulli's method, treated such knowledge as a gainful property to be guarded by secresy.

"Liverpool, London, and other places, had their tide tables constructed by undivulged methods, which methods, in some instances at least, were handed down from father to son for several generations as a family possession, and the publication of new tables, accompanied by a statement of the mode of calculation, was resented as an infringement of the rights of property."—*Whewell's Philosophy of the Inductive Sciences II, p.* 256.

The Tide Tables first put into circulation for the port of Liverpool were those originated by the Rev. George Holden, of which, he tells us, the theory was supplied by his brother, the arithmetical calculations by himself. These tables were first issued in 1770, and contained the *times* of high water, but not the *heights*. With regard to the heights, he remarks in the introductory address, "the heights, I am persuaded, may be as truly computed as the times, but I cannot insert them for want of having some equations truly determined." Subsequently, however, the heights were inserted, and the diurnal inequality professedly allowed for ; but the principles of the method have never been published. Hence the method cannot be criticized, but the results shew one of two things—either a mistake in the principles, or the omission of some element in the calculation ; since, although in general they are sufficiently accurate, in certain instances they are affected by a persistent series of errors.

Tide Tables for Liverpool are also published annually in the Isle of Man ; and, as the calculations are performed independently, the results at times vary considerably from those of Holden.

It remains now to give some account of the Harmonic Analysis of the Tides, introduced by Sir William Thomson, and carried out into figures by Mr. E. Roberts, of the Nautical Almanac Office. A few remarks only are necessary, inasmuch as the Reports of the British Association dwell upon this method exclusively, and appear to take for granted that any modification of the Equilibrium-theory, which shall extend its details and bring it into nearer agreement with the facts of observation, is simply impossible. The author of the Harmonic Analysis system even went so far as to observe "that he did not think it would be advisable to print any tables regarding this subject which are founded on any method less complete than that of the Harmonic Analysis employed by the British Association's Tidal Committee." Surely, in this case, it is not inappropriate to quote the language of a distinguished writer, who says—"Art, in this instance, having cast off her legitimate subordination to Science, has resumed her ancient practices of exclusiveness."

In this method the tide-wave, considered with reference to the time when it reaches a given point of the earth's surface, and to its height at that point, is made up of the superposition of a series of waves of different amplitudes and periods, arising from the different relative positions and varying distances from the earth of the disturbing bodies, the sun and moon, and of the variation of certain elements of their orbits. Of these waves, each of which has a typical analytical expression, twenty-three are distinguished—

Two	The lunar monthly and solar annual (elliptic).
Two	The lunar fortnightly and solar semi-annual (declinational).
Four	The lunar and solar diurnal (declinational).
Two	The lunar and solar semi-diurnal.
Seven	The lunar and solar elliptic diurnal.
Four	The lunar and solar elliptic semi-diurnal.
Two	The lunar and solar declinational semi-diurnal.[*]

An instrument called a "Tide Predicter" has been constructed for the purpose of computation, which mechanically delineates the tidal curves from day to day, and indeed from instant to instant. It appears, however, to the author of these pages, that in order to fulfil the task assigned, two such machines must be employed, the one for lunar, and the other for anti-lunar tides, as will be explained in a subsequent chapter.

The labours of Professor Haughton, F.R.S. of Trinity College, Dublin, must next be alluded to. A series of Tidal Observations made in 1850 and 1851, under the direction of the Royal Irish Academy, were discussed and published by him, so far as relates to the Diurnal Tide, in 1854. These observations were conducted at ten ports in Ireland during twelve months, and with them were obtained corresponding results of the Semi-diurnal Tide. As general results deduced from these observations we have—

---

[*] Admiralty Manual of Scientific Enquiry, page 82.

Ratio of Solar and Lunar Coefficients .... = 0·3353
Mass of Moon... = $\frac{1}{60{\cdot}35}$
Mean Depth of Sea from Tidal Intervals .... = 3·434 miles.
Eccentricity of Moon's Orbit .... = 0·0632
Mean parallactic range from South Channel Tides .... = 2·6 feet.

Professors Haughton and Galbraith have also issued a compendious manual on Tides and Tidal Currents, which has obtained a wide circulation. The former has also reduced the Tidal Observations made during the Arctic Voyages of Sir Leopold McClintock and others; and much useful information may be gathered from his papers, which are published in the Transactions of the Royal Irish Academy.

CHAPTER II.

THE GENERAL THEORY.

THE subject of the Tides is one which especially forces itself upon the attention of those who reside near the sea-coast. It is one, however, upon which people are content with general and often inaccurate information, notwithstanding that the details of it are not difficult, but exceedingly interesting and instructive. Most persons are aware that the tides are caused by the joint attractions of the moon and sun, these attractions being more visibly exerted upon the water than upon the land, simply in consequence of its fluidity. When the attractions of the moon and sun pull in the same line, or nearly so, we have "spring tides," and when they pull at right angles to each other's direction we have "neap tides."

These circumstances will be at once understood by referring to the illustrative diagrams. In fig. 1, S represents the sun, M the moon when new, and M′ the moon when full. The solar and lunar tidal waves in both these cases coincide, and are heaped upon each other, producing what are called SPRING TIDES.

In fig. 2, S represents the sun, and M and M′ the moon in the quarters. In this case the solar tidal-wave is ninety degrees, or a quarter of the earth's circumference, from the lunar tidal-wave. It is evident, therefore, that since, in this case, the solar tide is formed at the expense of the lunar, the lunar tide will be of diminished height. The tides in this case are called NEAP TIDES.

'The cause of the tides, however, as far as the action of the moon is concerned, is not, as is commonly supposed, due to the mere attraction of the moon, since if that attraction were equal in all the component parts of the earth, there would assuredly be no tides at

PLATE I

FIG I

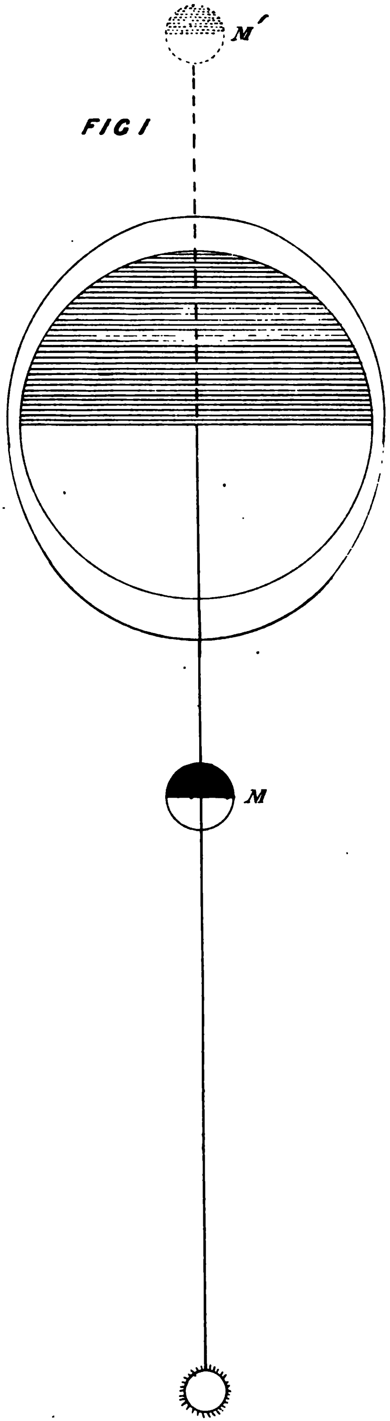
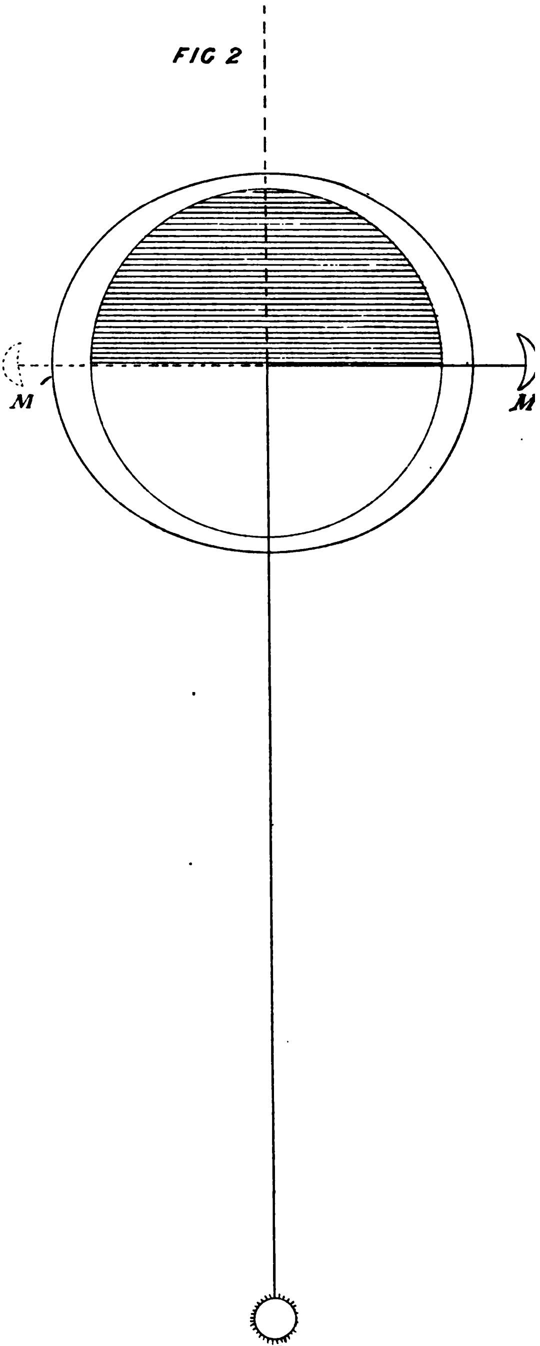


FIG 2



all. We are to look for the cause, not simply in the attraction of the moon, but in the *inequality* of its attraction on different parts of the earth. Hence, as the moon is subject to a slight variation of distance from the earth, it will follow, that when the moon is at its least distance, or at a point called *perigee*, the tides will be greatest; and when the moon is at the greatest distance, or at a point called *apogee*, the tides will be least; not simply because the entire attraction of the moon in the former case is greater than in the latter, but because the diameter of the globe bearing a greater proportion to the lesser distance than to the greater, there will be a greater *inequality* of attraction.'

'The attraction, then, which the moon exerts upon the shell of water which is collected immediately under it is greater than that which it exercises upon the solid mass of the globe, so far, that is, as that attraction is influenced by distance: consequently there will be a greater tendency to draw the fluid which rests upon the surface towards the moon, than to draw the solid mass of the earth which is more distant; and as the fluid is, by its nature, free to obey this excess of attraction, it will necessarily heap itself up in a convex protuberance, or, as it is sometimes called, a wave, on that side of the globe nearest to the moon. The water thus collected will necessarily flow from regions in the circumference at right angles to the line joining the earth and moon, where, therefore, there will be a diminished quantity of water in the same proportion.'

'But now let us consider what happens to that part of the earth most remote from the moon. Here the waters, being more remote from the moon than the solid mass of the earth under them, will be less attracted, and consequently will have a less tendency to gravitate towards the moon. The solid mass of the earth will, as it were, recede from the waters farthest from the moon, in virtue of the excess of attraction, leaving those waters behind it, which will thus be heaped up so as to form another convex protuberance on the remote side, similar exactly to that already described as produced on the nearer side. As the difference between the attraction of the moon on the nearer waters and the solid earth under those waters is nearly (though not quite) the same as the difference between its attraction on the solid globe and the more remote waters beyond it, it follows that the height of the fluid protuberances will be nearly (though not quite) equal. In other words, the heights of the tides on opposite sides of the earth, the one being under the moon and the other more remote from it, are nearly (though not quite) the same.'*

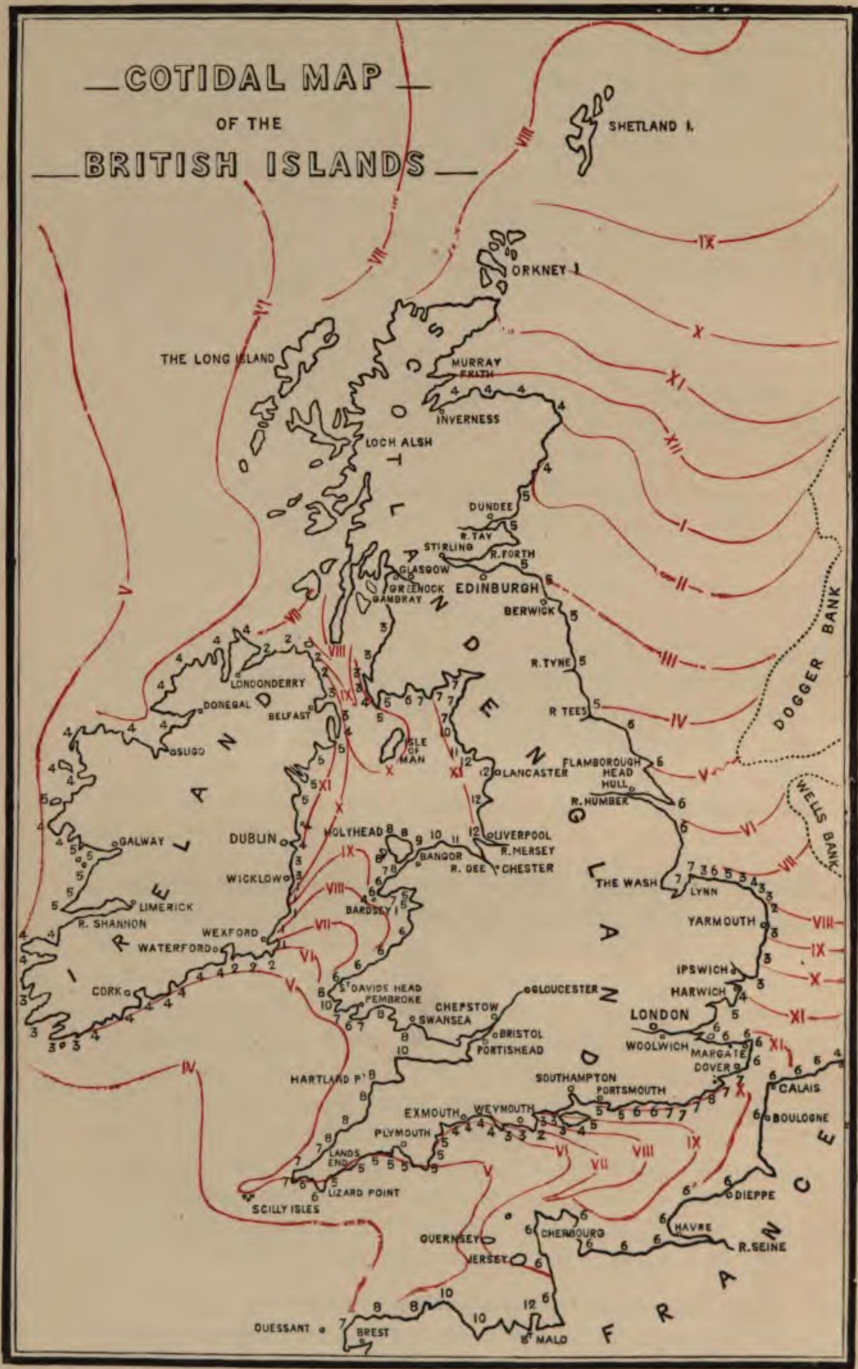
*Lardner's Museum, vol. 7, p. 132.

We may illustrate this part of our subject as follows:—Suppose three persons, A, B, C, to be standing 10 feet apart from each other, and let A be pulled forward with a force (say) of 12 lbs, B with a force of 10 lbs, and C with a force of 8 lbs. Then A, under the action of the stronger force, will move forward to a distance (say) of four feet, in the direction of the force; B, under a less force, to a distance (say) of two feet; and C, under the least force, to a distance (say) of one foot. Consequently A's distance from B is now no longer 10 feet, but 12 feet; whilst C's distance from B is no longer 10 feet, but 11 feet; that is to say, both A and C are at a greater distance from B than they were before. The very same sort of thing takes place in the waters of the ocean, and hence we have two tides at the same instant on opposite sides of the globe, which by the rotation of the earth are carried round to its various coasts at intervals of twelve hours.

It will doubtless occur to those who bestow attention to these observations, that all we have stated in reference to tides produced by the moon will also be applicable to tides produced by the sun. Hence arise four sets of tides—*lunar*, *anti-lunar*, *solar*, *anti-solar*. When any two of these four coincide, one of the two being due to the moon and the other to the sun, so that the waves are superimposed, we have *spring tides;* when their crests are as wide apart as possible, or ninety degrees asunder, we have *neap tides*. The moon's power to raise a tide is greater however than that of the sun in the proportion of five to two; for, though the mass of the sun is 28,394,000 times that of the moon, it is at a distance from the earth 400 times greater.

'If physical effects followed each other immediately without any appreciable interval of time for the operation of their causes, then the tidal-wave produced by the moon would be found on that meridian of the earth which is directly under and opposite to the luminary, and the same would be true of solar tides. But by the inertia and friction of the waters this is prevented, and a retardation is produced. In the first and third quarters of the moon the solar tide is westward of the lunar, consequently the actual high water, which is the result of the combination of the two waves, is westward of the place where it would have been if the moon had acted alone. In the second and fourth quarters the solar tide is eastward of the lunar, and the general effect produced by the sun and moon combined gives rise to what is called the *priming* and *lagging* of the tides.'

'The apparent time of high water on the day of new or full moon is what is usually called the *establishment* of the port. Dr. Whewell



COTIDAL MAP OF THE BRITISH ISLANDS

calls this the *vulgar establishment*, and he calls the *corrected establishment* the mean of all the intervals of the tides and transits of half a month. The time elapsed between the original formation of the tide and its appearance at any place is called the *age* of the tide, and sometimes, after Bernoulli, the *retard*.'

But in the next place, let us direct our attention to the actual course and progress of the tide-waves on the day of new or full moon, when the action of the two luminaries, the moon and the sun, may be considered as in the same straight line. On that day there is formed, under the moon's eye as it were, a huge tide-wave, whose crest has its position chiefly in the South Indian Ocean, where a zone of water exists of sufficient extent to allow a line of contemporary tides, or *cotidal-line* as it is termed, to be fully formed. At the same instant a like tide is being developed in the opposite hemisphere by the obverse action of the moon. Each of these tide-waves circulates round the globe, bringing high water twice a day; and each of these on passing the South African promontory sends a derivative undulation up the Atlantic Ocean. This derivative undulation arrives midway up that ocean twelve hours later, and it keeps always a certain proportion of its original magnitude, direction, and velocity. In travelling along this ocean the wave assumes a curved form, the convex part keeping near the middle of the ocean and ahead of the branches, which, owing to the shallower waters, lag behind on the American and African coasts, so that the cotidal lines have always a tendency to make very oblique angles with the shore, and, in fact, run parallel to it for great distances. Seven hours later, *i.e.* at five o'clock on the morning of the following day, this derivative wave has reached simultaneously the entrance to St. George's Channel to the south, and the entrance to the North Channel to the north, having been broken into two waves by the long coast-line of Ireland. Thenceforward these two waves approach each other, and finally meet and blend in the middle of the Irish Sea.* The velocity of the waters being thus checked, the blended waters are diffused over the flat areas of Morecambe Bay, Solway Frith, and the entrances of the Ribble and Mersey, and thus they conspire with the rivers in producing the vast sandbanks peculiar to those localities. The same waves also bring high water to the east coast of Ireland. The waters, on leaving, again betake themselves to the entrances of the channels from which they started, and there help to build up another advancing wave, which goes through the same circuit, and so on continually. But all this while the main body of the tide-wave has proceeded onwards towards the Arctic regions, the portion we have considered being merely its margin. In like manner, also, the English Channel has been traversed by another branch of the same derivative tide-wave, which meets, at or near the Straits of Dover, another tide-wave, which has circulated round the north of Scotland and down the North Sea, having started from the main body of the tide in the Atlantic at a period twelve hours earlier.

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*For tides of the Irish Sea, consult papers by Rear Admiral F. W. Beechey.

From what has been said, it will be noticed that when it is high water at the entrances of the Irish Sea, it is low water in the middle of that Sea, and *vice-versâ*. Hence the level of the waters oscillates with a kind of "see-saw" motion, and at the middle of the "see-saw," which crosses St. George's Channel opposite Courtown, there is little rise or fall of tide. This line is called a "*nodal-line*" or "*hinge*" of the tide, and there is another nodal-line north of Belfast. So also there are nodal-lines in the North Sea and in the English Channel. When, however, we speak of the motion of a tidal-wave, it must not be imagined that the mass of water of which the wave is composed has this velocity. The motion of the tidal-wave is only a particular instance of undulatory motion, a motion of *form*. A ship floating upon the sea is not carried forward by the progressive waves, but simply rises and falls as they advance and retire.

Having thus shewn that the time and height of any tide depend on the times of transit, position, distance, velocity, and direction of motion of the moon and sun, we pass on to consider how all these peculiarities may be practically utilized in the construction of tables of computation. With this object Sir John Lubbock distinguishes the successive transits, whether upper or lower, of the moon, which form what is termed the argument in many of his tables, by the successive letters of the alphabet—A, B, C, D, E, and F. Thus, on full or change days, for instance, if E denote the time of the moon's transit at Greenwich, which follows next after the time of any high water at Liverpool, D will denote the time of transit, upper or lower, next preceding, *i.e.* about 12½ hours earlier, at which time the tide-wave which brings that high water has its crest in the middle of the Atlantic; C will represent the time of transit next but one preceding, at which time the said tide-wave has its crest in the South Atlantic, circulating round the Cape of Good Hope; whilst B will fix the time of transit next but two preceding, *i.e.* something more than 1½ days earlier, at which time the crest alluded to is in the vicinity of Australia. Or, to put it differently, if

A denote the time of transit on Monday morning,
B will denote the time of transit on Monday afternoon,
C " " Tuesday morning,

D will denote the time of transit on Tuesday afternoon,
E " " Wednesday morning,
F " " Wednesday afternoon.

Some such distinction of the transits is obliged to be introduced, both in order that the transit from which the establishment of the port is taken may not be left in ambiguity, and also in order that what is termed the *semi-monthly inequality* may be correctly obtained. It is found that transits A or B are equally convenient, but one or other must be selected. The *semi-monthly inequality* in the times or heights of high water exhibits all the mean values of those quantities, as shewn in the course of a semi-revolution of the moon round the earth. The semi-monthly inequality in the times deduced from observation, *when the proper constants are employed*, is identical with that obtained from the mathematical expression as worked out by Bernoulli. The like is true as regards the heights, but the epoch appears to be different from that which is employed when the times are being calculated. The tables so formed are given at the end of this work—Tables I and VI. Certain other tables follow, shewing the corrections to be added to or subtracted from the numbers in the fundamental table in consequence of the changes in moon's and sun's distance, declination, &c., presenting a degree of regularity which is to a certain extent artificial; but there are modifications introduced from numerous and careful observations which tend much to promote their accuracy.

In making use of the factor by which the height of the tide at any port is connected with the height at any other port, it is necessary to observe that the result must only be regarded as a first approximation. In order to obtain more correct results, the course and magnitude of the semi-monthly inequality ought to be ascertained for each port by an independent determination. To accomplish this we must have recourse to a method in which each tide is separately noted, and assigned to its own classification throughout a year at least. It will be found that the heights of high water at different places do not differ by a constant quantity, and that the semi-monthly inequality has not everywhere the same form. The statement is again repeated, that, though the form or law of the semi-monthly inequality, both in the interval between the moon's transit and the height of high water, seems to agree with the formula given in mathematical language by Bernoulli, when the proper constants are employed; yet this agreement must be received with some limitations, for while the semi-monthly inequality in the interval would lead us to suppose that the tide is due to the attractions of the luminaries at the time of the transit A, the inequality of the height seems in certain cases to be due to the attractions of each of the luminaries at the time of the transit B. For this and other reasons, a separate investigation of the semi-monthly inequality at each port becomes necessary; and in the exceptional cases of the estuaries of the Thames and Severn, where tides of different ages arrive simultaneously, an additional difficulty is encountered, which requires an independent investigation in order to deal with it.

CHAPTER III.

THE NEW MODIFICATIONS.

IT has been already observed that the elevation of the tides is due, not only to the action of the moon and sun on the waters immediately beneath them, but also that this action is extended directly through the solid body of the earth to the waters on the obverse side of it. The effect may in fact be conceived by supposing two moons, one on each side of the earth, the line joining them always passing through its centre, and each attracting so as to produce on its own side the tide actually exhibited. The first of these being in the place of the real moon, the second or fictitious one may be called the "anti-moon."* In the very same manner we must imagine the tides which are produced by the attraction of the sun to be of the same twofold character. We are thus introduced to four sets of tides, operating, as will be seen, under extremely different circumstances, namely—1st a *lunar* tide, swelling under the direct action of the moon; 2nd an *anti-lunar* tide, swelling under the obverse action of the moon, or, as we may put it, the direct action of the "anti-moon"; 3rd a *solar* tide, generated under the direct action of the sun; and 4th an *anti-solar* tide, due to the direct action of the "anti-sun." It is most important clearly to comprehend these distinctions, since each of the tides so considered has its own peculiar properties, its own peculiar direction and velocity of motion. It is evident also that when the true moon or sun is over the northern hemisphere or has "north declination," the fictitious "anti-moon," or "anti-sun" is over the southern hemisphere or has "south declination." More than this, it will be seen that as either of these luminaries advances from south to north declination, its fictitious antagonist will recede from north to south declination, and *vice versâ*. If a velocity of angular rotation equal and opposite to that of the earth be supposed to be communicated to the moon, the earth will be reduced to rest, and the moon's path in the sky will be represented by a sort of spherical *helix*, the convolutions of which will ascend from east to west, whilst the moon's declination passes from south to north ; and during that interval, the path of the "anti-moon" will be also represented by a corresponding *helix*, but differing from the former in this respect, that its convolutions will descend from west to east whilst the actual moon's declination has the fore-mentioned changes. To use a familiar illustration—in the one case we shall have a cork-screw motion from left to right, and in the other from right to left. Consequently the lunar and anti-lunar tides are essentially different in their operation, and so are the solar and anti-solar tides. But forasmuch as land predominates in the northern, and water in the southern hemisphere, the magnitude of the tides produced will from this cause alone be very different, and when the resulting tides which are formed of the super-imposed tides, due to the action of the moon and sun, reach our shores, they will display the difference of magnitude very prominently. This irregularity is called the "diurnal inequality," and it has hitherto been the fruitful source of the discrepancy between theory and observation.

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*This device is due to Laplace.

In order to make this matter more intelligible, let us suppose the earth entirely covered with water to an uniform depth, and the moon alone existing as an attracting body. Theoretically, under these circumstances the action of the moon produces a tidal spheroid, and the protuberances of this spheroid may be considered as constructing two tidal-waves—one following the moon, and the other opposite to this. When the moon is not in the equator, one of these may be called the northern, and the other the southern tidal-wave. Now, places in the northern hemisphere pass nearer to the pole of the northern tidal-wave, and places in the southern hemisphere pass nearer to the pole of the southern tidal-wave, and these tides are alternately greater and less. Any place, therefore in the northern hemisphere would be washed by two tides daily, but alternately by that tide whose crest is in the northern hemisphere, and by that whose crest is in the southern hemisphere, and these tides would be of different magnitude from this cause. In other words there would be a "diurnal inequality." The like will of course be true when the sun is the only luminary considered. This is precisely what the hitherto-received theory advances (see Whewell's Dynamics, part i.

p. 198), and I am not aware that the more modern theory, which is based upon the Harmonic Analysis of the tides, contradicts the more ancient one. But in point of fact, and as a matter of observation, the configuration of land and water on the globe is such as to cause the tide whose crest is (or rather ought to be) in the northern hemisphere, to be invariably of lower magnitude than that whose crest is in the southern hemisphere; and this from the simple reason that neither moon nor sun can possibly generate a tide in regions where little or no water exists for the purpose.

Besides this, the *direction* of the moon's motion makes a very great difference in the magnitude of the tide-wave which reaches our shores. For, when the moon advances from south to north declination, crossing the equator, as she sometimes does, at an angle of 28 degrees, and sometimes at 18 degrees,* it is seen that her line of motion approximately coincides with the general trend of the Atlantic Ocean, at the time the Earth's rotation brings that part of the globe directly beneath her; and this causes a further development of the tide-wave in the direction of Europe and North America, whereas, when the moon declines from north declination to south, her course is diagonal to the former one, crossing the Atlantic, roughly speaking, in the direction of its *breadth*, whilst in the other case she crossed in the direction of its *length*. The like phenomena apply to the "anti-moon," the sun and "anti-sun," when these are the agencies considered. And, in point of fact, this configuration of land and water produces peculiarities in the circumstances of each port which cannot be ignored, when its special tide-tables have to be calculated.

It is plain, then, that a complete distinction exists in the mode in which the tide-generating forces operate in producing the two tides of each successive day, and if it were possible that the times and heights of continuous tides could be worked out by machinery, as proposed by the "Tide Predicter" of the Harmonic Analysis, two such machines would have to be employed, the one for lunar and the other for anti-lunar tides. But even these would be insufficient, because the distinction between the solar and anti-solar tides would be likewise involved. In a free ocean everywhere surrounding the earth, the case would be wholly different. In dealing with the problem as we find it, we have to employ a tentative method, analogous to that used in attempting the solution of an inverse problem. The tides caused by the moon and sun are fourfold, but they circulate round the globe in widely different courses, and with differing angles of impact on the various shores or coast-lines which intercept their progress. In the newly-devised method of dealing with the subject, any single tide during the year will have only one tide corresponding to it out of all the tides in any other year; and hence it becomes interesting to enquire how far *correlative* tides (as I may term them), that is, tides having the same, or nearly the same constituents, agree with observation as years advance; for as like causes produce like effects in nature, the resulting tides ought to be nearly the same in such cases. There are, however, certain causes of discrepancy, which must not be lost sight of; and amongst these the foremost are the pressure of the atmosphere, as indicated by the height of the mercury in the barometer; and the magnitude and direction of the gradients belonging to the depressions and elevations, shewn in the weather charts which are published daily by the Meteorological Office in London. Another disturbing element is the *suddenness* with which the pressure shifts its direction. Thus a south-east wind rapidly changing into a south-west wind causes an unusual elevation in the level of the Irish Sea. The pressure of the atmosphere amounts to nearly a ton on every square foot of the surface of the ocean, and the rise or fall of an inch in the barometer denotes an increase or decrease of pressure amounting to upwards of 5½ cwt. on every square yard; and the waters being inelastic, the effect is similar, in the case of a contracted channel such as that which leads to the Irish Sea, as that which would be produced if a lid were made to cover the upper surface of a long wooden trough filled with water, the lid being hinged at its middle points, and a pressure being applied to one end.

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*This cycle of changes accounts for the periodical encroachment and withdrawal of the sea, as well as for the shifting of sandbanks.

In order then to compare *correlative* tides, we shall select a series occurring in the same month (April) in each year subsequent to 1872, and it will be seen that those which have nearly the same constituents, when the atmospheric conditions are the same, give resulting tides which agree with observation; and when those constituents are diverse there is a corresponding discordance. The like is shewn in a second series selected in the same month (September) in different years. For instance, the tides of September 15th, 1872, evening, and September 5th, 1873, evening, have nearly identical constituents, and yet from atmospheric causes, we have in one case a tide three inches above calculation, and in the other a tide nine inches below.

It needs scarcely be added that the same atmospheric conditions which tend to diminish the height of a tide, make the ebb to be lower to the same extent, and *vice versâ*.

COMPUTATION OF TIDES.

Anti-lunar and Solar.	1874, April 26, Morning.		1875, April 15, Morning.		1876, April 4, Morning.		1877, April 23, Morning.		1878, April 13, Morning.	
	h. m.	ft. in.	h. m.	ft. in.	h. m.	ft. in.	h. m.	ft. in.	h. m.	ft. in.
Moon's Transit	7 25	21 2	7 7	20 9	7 16	20 11	7 12	21 0	7 18	21 0
Dec. for Anti-lunar ...		— 1		— 1		— 1		— 1		— 1
Moon's Hor. Parallax..	55′ 0″—	— 5	57′ 17″—	+ 5	58′ 55″—	+14	59′ 21″ +	+14	59′ 0″+	+12
Moon's Declination ...	19° 55′ S. asc.	0	24° 2′ S. asc.	— 6	25° 2′ S. asc.	— 8	12° 28′ S. asc.	+ 3	19° 43′ S. asc.	+ 0
Sun's Declination	13° N.	+ 2	9° N.	+ 1	5° N.	+ 1	12° N.	+ 1	8° N.	+ 1
	ft. in.		ft. in.		ft. in.		ft. in.		ft. in.	
	20 9 obs.	20 10	20 8 obs.	20 8	21 7 obs.	21 5	22 2 obs.	22 5	22 2 obs.	22 0

Lunar and Anti-solar.	1872, Sept. 15, Evening.		1873, Sept. 5, Evening.		1874, Sept. 24, Evening		1875, Sept. 13, Evening.		1877, Sept. 20, Evening.	
	h. m.	ft. in.	h. m.	ft. in.	h. m.	ft. in.	h. m.	ft. in.	h. m.	ft. in.
Moon's Transit	21 37	24 5	21 48	24 8	21 56	24 9	21 34	24 5	21 48	24 8
Inc. for Lunar		+ 5		+ 5		+ 5		+ 5		+ 5
Moon's Hor. Parallax..	60′ 25″+	+20	60′ 27″+	+20	59′ 36″+	+15	56′ 27″+	+ 0	54′ 2″+	—12
Moon's Declination ...	21° 22′ S. asc.	— 2	23° 24′ S. asc.	— 4	17° 23′ S. asc.	+ 3	22° 1′ S. asc.	— 3	15° 22′ S. asc.	+ 4
Sun's Declination	4° S.	+ 5	7° S.	+ 4	0° S.	+ 5	5° S.	+ 5	2° S.	+ 4
	ft. in.		ft. in.		ft. in.		ft. in.		ft. in.	
	27 0 obs.	26 9	25 10 obs.	26 9	27 1 obs.	27 1	24 8 obs.	25 0	24 7 obs.	24 9

It may be of interest to place on record examples of the most extreme cases of high and low tides which I have as yet observed. The tides in these instances were produced, of course, under circumstances least favourable and most favourable to their development.

	1876, March 19, Evening, Anti-lunar and Anti-solar.		1874, March 19, Evening, Lunar and Solar.	
	h. m.	ft. in.	h. m.	ft. in.
Moon's Transit	18 8	20 0	11 55	26 3
Inc. for Lunar or Anti-lunar,		— 4		+ 7
Moon's Horizontal Parallax.	54' 14"—	—12	61' 22"+	+27
Moon's Declination	28° 44' N asc. des.	—19	5° 27' S. asc.	+14
Sun's Declination	1° S.N.	— 3	0°	+ 7
	ft. in. 15 8 obs.	16 10	ft. in. 30 11 obs.	30 10
	(Helbrè Island.)			

In the former case the tide fell fourteen inches below its expected height; but the barometer on this occasion stood at 30·7 in., and the wind was very cold and heavy from the north. Such a low tide had not been remembered for at least twenty years.

If, then, we combine all the results which follow from these considerations we shall obtain at once an insight into the rationale of the following facts, which a careful observation of the tides of the Irish Sea abundantly confirms.

1.—During the 1st and 4th quarters of the moon the lunar and solar tides are super-imposed so as to form one tide, and the anti-lunar and anti-solar are super-imposed so as to form the succeeding one.

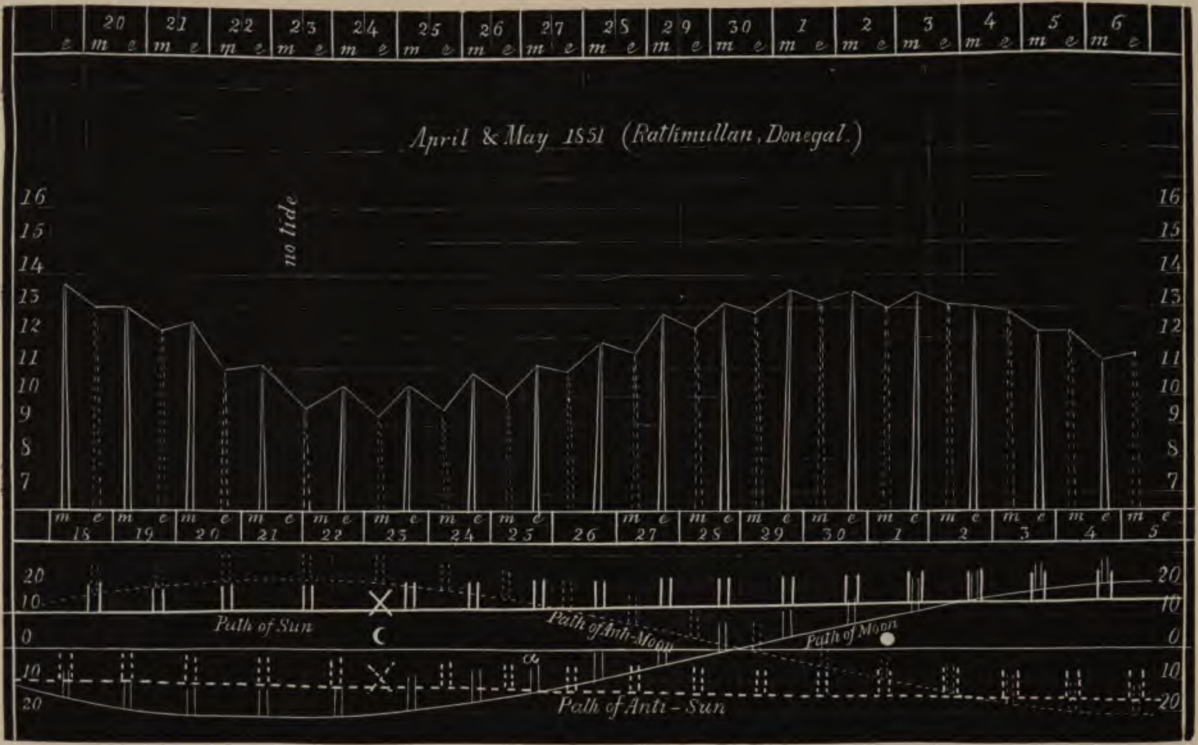
2.—During the 2nd and 3rd quarters of the moon, the lunar and anti-solar tides are super-imposed so as to form one tide, and the anti-lunar and solar tides are super-imposed so as to form the succeeding one.

3.—The tide-generating influence of the moon is greatest when her declination ranges from 20 deg. S. *ascending* to about 15 deg. N. *ascending*, and it is least when her declination ranges from 20 deg. N. *descending* to about 15 deg. S. *descending*. The like is true of the anti-moon, *mutatis mutandis*, the parallax being the same in both cases.

4.—The tide-generating influence of the sun or anti-sun is greater when the declination is S. than when it is N.

5.—There is least "diurnal inequality" when the moon's declination ranges beyond 15 deg. whether N. or S.

It is desirable to add a few words on the best method of noting and recording the successive heights of the tides. The accompanying



April & May 1851 (Rathmullan, Donegal.)

graphic process has the merit of exhibiting at a glance, not only their elevations, but also the designation of each tide according to the remarks already made, and the various causes which combine to give rise to it. The diagram requires some explanation. The figures at the side denote the successive feet as marked on an Admiralty tide guage, whose base is the mean level of low water of ordinary spring tides. The figures at the top of the page point out the days of the month (April) during which the observations are made, and the letters "m, e," refer to the morning or evening tide, as the case may be. The figures at the foot of the diagram refer also to the days of the same month, but each of them put a day and a half antecedent to those in the top line, because the tide takes that interval in travelling from its cradle in the South Pacific to the Irish Sea. The tides are marked alternately by dotted, and by straight lines, the latter indicating what we have called "lunar," and the former "anti-lunar" tides, and they are laid down to scale, thus shewing, by the lines joining the summits the course of the curve they form. Within the space at the lower part of the diagram, the moon's path, and that of the sun are plotted also to scale, and the dotted lines show those of the "anti-moon" and "anti-sun." A glance from the top to the bottom of the diagram, in the direction of the lines which mark each tide, enables us to complete our description. If we take the morning tide of April 25th, for instance, we can see that it is generated partly by the direct action of the moon with the declination south *ascending* (a circumstance which is favourable to its development), and partly by the direct action of the sun with declination north. The evening tide, on the contrary, is connected with the action of the "anti-moon" in the northern hemisphere, and that of the "anti-sun" in the southern. The former, therefore, is a *lunar* tide compounded with a *solar*, and the latter is an *anti-lunar* tide compounded with an *anti-solar*. The positions of the moon at each quarter are likewise noted, and also at apogee and perigee, and thus it is seen how the hour-angle and the moon's distance separately affect the height of each tide.

A few remarks are added in reference to the Tables which are appended at the end of this volume, and some of which are published for the first time. Of these Tables I—VI are identical with those which appeared in the Introduction to the Admiralty Tables in 1872, with the exception of the Liverpool column in Table VI, which has been slightly modified. Table VIa exhibits the excess of the lunar over the anti-lunar tide. It is found that, as might be expected, the lunar attraction, when operating on the nearer hemisphere of the earth's surface, is more powerful than when operating on that which is more remote. The earth's diameter, 8,000 miles, is about one-thirtieth part of the moon's distance, and in Liverpool, where equinoctial tides have a range of 30 feet, the law of gravitation, which lays down that the attraction varies inversely as the square of the distance, plainly asserts itself. The amount of this excess, however, in consequence of the configuration of land and water, depends on the season of the year, and the time of the moon's transit B.

Table VII shews the correction in height for the moon's parallax, the effect of which is maintained for some time after that parallax has reached its maximum or minimum. The time of the moon's transit B affects this correction also.

Table VIII shews, in a very abridged form, the correction for lunar or anti-lunar declination. It is in the construction of this table that the principles of the newly-devised method become most manifested. These have been already expounded, and the table gives a practical exemplification of their working. The figures have been noted after very careful observation.

Table IX is perhaps the most intricate in the group. It gives an interpolation for the effect of the combination of a solar or anti-solar tide with either of those produced by the direct or obverse action of the moon. If the sun were the only attracting body, the tides produced by its action would arrive on our shores at the same time or nearly so, morning and evening, throughout the year. In winter, at Liverpool, the solar day-tides would be highest, in summer they would be lowest. They are, however, blended with the lunar tides; but in forming this table they have to be distinguished, and the mode in which they interfere has to be carefully traced and tested.

CHAPTER IV.

MATHEMATICAL INVESTIGATIONS.

WHEN searching for an explanation of the cause of the tides on mechanical principles, it readily occurred to those philosophers who had familiarized themselves with the theory of Universal Gravitation, to connect the phenomena with the sun and moon, as influencing by their attraction the waters of the ocean. Accordingly various theories have been adopted for the calculation of the tides on this hypothesis, of which the most noted have been those of D. Bernoulli and Laplace. "If the hypothesis of universal gravitation be adopted, there can be but one correct theory based upon it for calculating the oscillations of the ocean; but in consequence of the difficulties of the analysis, which have hitherto been insurmountable, other hypotheses (or rather modifications) must be adopted in order to obtain an approximate solution of the problem. The irregularity of the depth of the ocean, the manner in which it is spread over the earth, the position and declivity of the shores, and their connections with the adjoining coasts, the currents, and the resistance which the waters suffer, cannot possibly be subjected to an accurate calculation, though these causes modify the oscillations of the great fluid mass. All that can be done is to analyse the general phenomena, which must result from the attractions of the sun and moon, and to reduce from the observations such data as are indispensable for completing in each port the theory of the ebb and flow of the tides."*

Sir J. Lubbock, in his Elementary Treatise, has given some of the mathematical investigations from which theoretical results have been computed, but as the present work is not intended for advanced students, and as such details are by no means interesting to the general reader, it has been thought best to omit them. The computation on Bernoulli's hypothesis proceeds on the assumption that the attraction of the moon causes the ocean to take at each instant the form it would have if the earth and moon were stationary. A prolate spheroid of revolution in a state of instantaneous equilibrium is imagined, its major and minor axes are assigned, and then solid geometry exhibits the mathematical relation between the length of a radius-vector at any point, and the angles which fix its position, the constants being determined from observation. The like also is done when the sun is the luminary considered. The excesses above the mean radii in each case are superimposed, and thus a formula for the height is obtained. A similar investigation brings out the interval between the time of transit and that of high water. As a final conclusion, the readiest method of calculating tables of heights and intervals theoretically, is to obtain the angle ψ (the hour-angle of the luminary at the time of high-water) from the expression—

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* Pratt's Mechanical Philosophy, p. 581.

$$\text{Tan } 2\psi = \frac{\text{A} \sin 2\phi}{1 + \text{A} \cos 2\phi}$$

for given values of ϕ (which represents the difference of their right ascensions); and then the height of high water$=\text{D} + \text{E} \left\{ \cos. 2\psi + \text{A} \cos. (2\phi - 2\psi) \right\}$.
where A and E are quantities involving the declinations and parallaxes, and D a constant depending on the zero-line from which the heights are calculated. Sir John Lubbock gives for their values, when moon's parallax$=57'$ and the sun's and moon's declinations are equal, log $\text{A}=9{\cdot}57848$, $\text{D}=8$ feet, $\text{E}=6{\cdot}969$ for Liverpool: with similar results for London. This theory is termed by Dr. Whewell the *Equilibrium Theory*.

'Laplace, however, takes a different course. He calculates the attractive forces of the sun and moon upon the ocean, and finds them to contain constant terms and periodical terms. He states that in consequence of the resistance and friction of the waters they would have soon assumed a form of equilibrium under the forces which are represented by the constant terms: and then, assuming this as a general dynamical principle, that the state of a system of bodies, in which the primitive conditions of the motion have disappeared by the resistance it suffers, is periodical when the forces themselves are periodical, he obtains an expression for the height of the tide, the same as that obtained from the Equilibrium Theory of Bernoulli. But there are so many assumptions in this, that we may, as far as we know, *à priori*, as readily adopt the Equilibrium Theory as that of Laplace.'*

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* Pratt's Mechanical Philosophy, p. 582.

PLATE IV

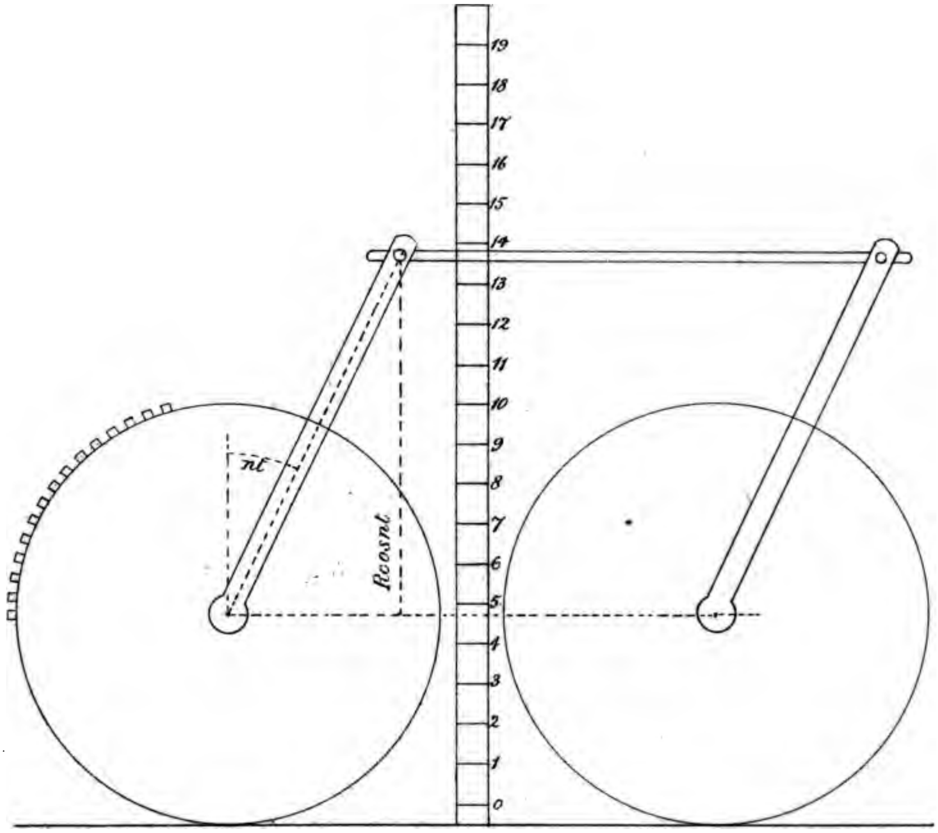

ILLUSTRATION OF A HARMONIC FUNCTION.

The following formula represents the height of a tide according to the Harmonic system,

$$h=A+(B\cos nt + b\sin nt) + (C\cos 2nt + c\sin 2nt) + \&c.$$

The numerical averages give 24 empirical values of h (from hour to hour) for each tide, and therefore there are as many equations of condition for determining the values of the constants A, B, b, C, c, &c.

In the Harmonic Analysis of the tides, the various changes of level to which the sea is subject, moment by moment, in consequence of the action of the tide-generating forces, are ascertained by the enumeration of a series of Harmonic Functions, each of which involves the time for which the computation is made, certain quantities depending on the angular velocity of the earth's rotation, the rates of relative orbital motion of the moon and sun, as determined by the lunar and solar theory, together with certain constants. What is called a Harmonic Function is an expression of the form R cos. $(nt-e)$, and the successive values it assumes throughout its cycle, may be shown by a simple contrivance. Suppose two wheels of the same diameter (like the driving wheels of a locomotive engine) with cranks attached to the centre of each, of equal length, and parallel to each other, the extremities of these cranks being held together by means of a connecting rod. Then, as is seen in the motion of a locomotive thus constructed, when the wheels turn round, the connecting rod will remain horizontal, but it will rise and fall above and below the line joining their centres once in each revolution, and its height above, or depression below this line, will be exactly R cos nt., when R is the length of each crank, and n the angular velocity of rotation of the wheel, *i.e.* the angle through which each turns in an unit of time. The rise and fall of this connecting rod, then, will represent the rise and fall of the sea during a tide, so far as that rise and fall depend on *any one* of the causes, or tidal constituents as they are called, which are many in number, by the sum-total of which the whole tide is generated. 'The chief tidal constituents in most localities are those whose periods are 12 mean lunar hours, and 12 mean solar hours respectively. These are denominated *semi-diurnal* tides. Those which probably stand next in importance have periods approximating to 24 hours, and are called *diurnal* tides. The semi-diurnal and diurnal tides have inequalities depending on the eccentricities of the moon's and earth's orbits, that is, on the parallax of each body; and the semi-diurnal have inequalities depending on the varying declinations of the two bodies, and the lunar semi-diurnal has also inequalities depending on the evection and variation perturbations of the moon. Each such inequality of any one of the chief tides may be regarded as a smaller superimposed tide of period, nearly, but not exactly, equal. The inequalities for parallax are called *elliptic*, and those for declination *declinational* tides: so that we have a number of tidal constituents or tides, and each of these adds to the whole effect observed, by supposing the observed height to be composed of the different heights due to the tidal constituents superimposed on each other.' There are also lunar fortnightly and solar semi-annual tides, as well as lunar monthly and solar annual tides, as enumerated in Chap. I.

The object of the instrument which has been recently devised for the mechanical computation of tides, and which has received the name of "Tide Predicter," is to effect the summation of the simple harmonic functions representing the various tidal constituents already referred to. The number of figures evolved in the arithmetical computation of such functions is simply enormous, and so in this case necessity seems to have been the mother of invention. The summation is performed by the machine in the following manner—'For each tidal constituent to be taken into account the machine has a shaft, with an overhanging crank, which carries a pulley pivotted on a parallel axis adjustable to a greater or less distance from the shaft's axis, according to the greater or less range of the particular tidal constituent for the different ports for which the machine is to be used. The several shafts, with their axes all parallel, are geared together so that their periods are to a sufficient degree of approximation proportional to the periods of the tidal constituents. The crank on each shaft can be turned round on the shaft, and clamped in any position: thus it is set to the proper position for the epoch of the particular tide which it is to produce. The axes of the several shafts are horizontal, and their vertical planes are at successive distances one from another, each equal to the diameter of one of the pulleys (the diameters of these being equal). The shafts are in two rows, an upper and a lower, and the grooves of the pulleys are all in one plane, perpendicular to their axes. The axes of the pulleys are set each at zero distance from the axis of its shaft at the commencement of the motion of the machine. A fine wire or chain, with one end hanging down and carrying a weight, passes alternately over and under the pulleys in order, and vertically upwards or downwards (according as the number of pulleys is even or odd) from the last pulley to a fixed point. The hanging weight consists of an ink-bottle with a glass tubular pen, which marks the tide-level in a continuous curve on a long band of paper moved horizontally across the line of motion of the pen by a vertical cylinder geared to the

revolving shafts of the machine. Hour marks are made on the curve by a small horizontal movement of the ink-bottle's lateral guides made once an hour, a somewhat greater movement giving a deeper notch to mark the noon of any day.'* Hence the machine indicates the heights of high and low water and the times of their occurrence, and the height of the tide at each instant. The curves on the roll of paper being drawn to scale, the times and heights have to be extracted by an ordinary graduated rule to feet and inches. A volume of tables for the ports of Bombay and Kurrachee was issued by Mr. Roberts for the year 1880, but the results of the comparison between theory and observation have not, so far as I am aware, been recorded.

The figure attached (which is taken from vol. 65 of the Minutes of the Proceedings of the Institute of Civil Engineers) 'represents the instrument in the stage up to which it is already completed, and in which the whole working of the composition of fifteen simple harmonic motions is in action, pulling up and letting down a weight, which in the completed instrument will be replaced by an ink-bottle. The cylinder for carrying the paper is not shewn in the drawing.'

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*Min. Inst. of Civil Engineers, vol. 65.

CHAPTER V.

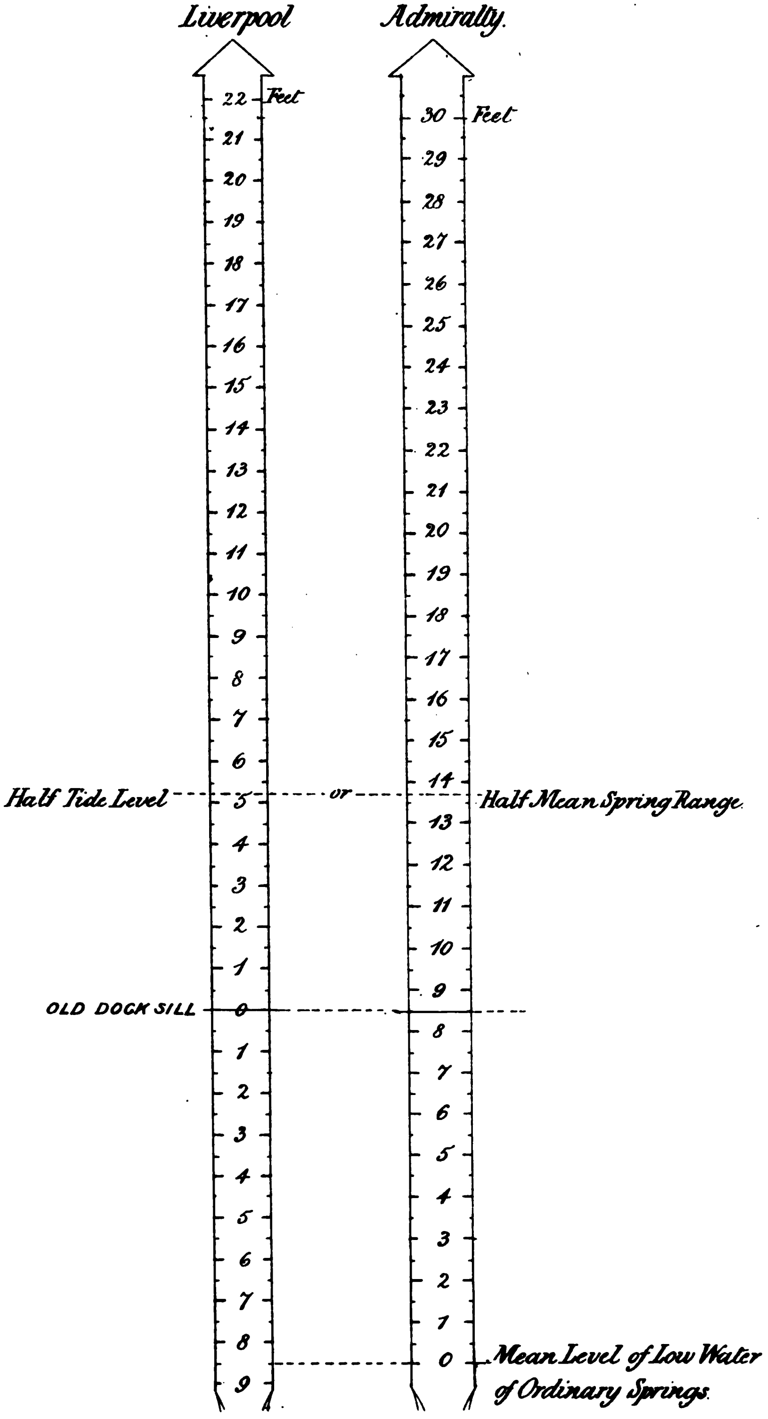
ON TIDE MEASUREMENTS, &c.

THE ever-varying height of the water-level in the sea-ports and harbours which commerce has created introduces us to a study, which is not only interesting and instructive, but imperatively necessary. A ship on arriving at her destination requires a safe access and convenient place of discharge, and these cannot be secured unless it is ascertained that a sufficient depth of water will be found to keep her afloat, and that a suitable time has been fixed upon for her entrance. A graduated scale therefore has to be erected, on which the height of the water can be marked in feet above a certain fixed base or datum-level. The selection of the position of such a base or datum-line is purely optional, but when once it is fixed upon, it cannot be altered without considerable inconvenience, inasmuch as the nautical charts of the port or harbour must recognize that position, or they would be valueless. For a similar reason it is necessary that the datum-level at one place should be compared and connected with that of every other: if not, the figures used, though true in themselves, might be deceptive. Thus a four-foot tide, measured from the mean level of the sea, if that were the base employed in one place, would mean something very different from a four-foot tide computed from the mean low-water mark, if that were used as base-line in another place.

When the Old Dock was first constructed at Liverpool, an epoch which may be considered as marking the commencement of modern navigation, it occurred to the merchants of that period to enquire simply how much in any instance the tide would rise over its sill, since that was the height which would tell them whether their ships could enter the dock or come out of it; and consequently the Liverpool Tide-Tables were first constructed merely to answer that

PLATE V

TIDE GAUGES.



question. The original dock no longer exists, but the ancient base-line—the Old Dock Sill—is retained on the east side of Canning Dock, and is still the ordinary line of reference in tables calculated for that port. The level adopted by the Admiralty is the *mean level of low-water of ordinary spring tides*, and hence appears a discrepancy in the figures which represent the heights, apparent only but not real. It is, for example, just as if the height of a room were measured from the top of the mantel-piece instead of from the floor,—the mantel-piece corresponding to the Old Dock Sill, and the floor to the mean level of low-water. The figures shewn would of course be smaller in the former case than in the latter.

It has been suggested that the line of half-range or half-tide would be a useful line of reference for levelling operations. In this case, however, a distinction must be observed between the mean-level of the sea and the mean-level of the half-tides in various places. The mean of the heights of high and low water is sufficient to determine the last, but a long series of observations become needful to discover the mean-level of the sea. And when discovered the position must be indicated by a permanent mark in stone or otherwise, just as that of the Old Dock Sill is marked on a marble slab inserted in the east wall of the Canning Dock. It has been found that the mean half-tide-level at Liverpool, above the Old Dock Sill, has increased continuously during successive periods of five years each, thus—

From 1854 to 1859 it was 4·948 feet
„ 1859 „ 1864 „ 5·073 „
„ 1864 „ 1869 „ 5·208 „
„ 1869 „ 1873 „ 5·265 „

It cannot, however, be supposed that the mean level of the sea has altered during these periods.

The selection of the half-tide-level of Liverpool as the datum for the Ordnance Survey of Great Britain was made after a series of tidal observations, carried on in 1859 at a number of ports round the coast of England.* In the Ordnance Book, "The Abstract of Levelling in England and Wales," it is defined in the following terms—"The datum-level for Great Britain is the level of mean tide at Liverpool, as determined by our own observations: it is $\frac{8}{10}$ of an inch above the mean tidal level obtained from the records of the self-recording tide-guage on George's Pier." These records give a total of 5·014 feet above the Old Dock Sill as the Ordnance Datum.

* See Report of British Association, 1875.

If then we assume 13ft. 9in. as the half mean spring range of ordinary tides at George's Pier, Liverpool, and 5ft. 3in. as the height of the level of this half mean spring range above the Old Dock Sill, we obtain 8ft. 6in. as the distance between the Old Dock Sill and the mean level of low water of ordinary springs at George's Pier, and these are the figures now adopted in the Admiralty Tables. At Helbre Island observation gives 9ft. 6in. as the corresponding distance in that locality: thus illustrating the fact that the configuration of coasts affects considerably the range of tide.

Another physical feature connected with the ocean level is deserving consideration, viz., the pressure of the atmosphere. On good authority we know that the height of high water in the English Channel varies inversely as the height of the barometer. The late Sir John Lubbock laid it down as a rule that a rise of one inch in the barometer causes a depression in the height of high water, amounting to seven inches at London, and to eleven inches at Liverpool. Sir James Ross, when at Port Leopold in the Arctic seas, found that a difference of pressure of ·668 inches in the barometer produced a difference of nine inches in the mean level of the sea—the greatest pressure corresponding to the lowest level. These results appeared to him to indicate "that the ocean is a water-barometer on a vast scale of magnificence, and that the level of its surface is disturbed by every variation of atmospheric pressure inversely as the mercury in the barometer, and exactly in the ratio of the relative specific gravities of the water and the mercury."*

It must be noted, however, that the motion of the sea, in rising and falling, requires more time than that of the mercury, owing to the great extent of surface influenced by atmospheric action.

In investigating this subject, it is necessary that the normal height of the tide should be correctly ascertained, *i.e.* the height under defined conditions of atmospheric pressure and of settled weather. Then the increment or decrement of the tide may be exhibited in the form of a supplementary table, using the height of the barometer and the direction and force of the wind as arguments. In this matter, however, experience is perhaps the safest guide.

A few observations may be added in reference to two subjects distinct from the theory of the tides as considered in these pages, yet not by any means unconnected with it. It is found that the earth's rotation-period, although practically of uniform length when tested by limited periods of time, is not really so when the time is still further extended; and this change in the length of this period is now suggested to arise from the friction of the waters which go to form the two great tidal-waves which constantly circulate round the earth. These waves travel in a direction contrary to the earth's motion of rotation, and therefore act as a kind of brake, whose tendency is to check it. The mere rise and fall in open ocean may not be strikingly indicative of this expenditure of force, but the consideration of it seems impressively brought before us when we see the behaviour of the tidal-waves in narrow channels, and when we admire heavily-laden ships carried majestically upon tidal rivers. The loss of rotation-motion is not great: the length of the day is said to be more by about one eighty-fourth part of a second than it was two thousand years ago.

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* Address of Sir F. J. Evans, K.C.B., Sect. E. Geography, British Association, 1878.

Another subject which has recently excited some attention is that of 'Tidal Evolution,' as enunciated by Mr. G. H. Darwin, Professor Purser and others. Dr. Ball, F.R.S., of Trinity College, Dublin, writes thus concerning it in the *English Mechanic* of Aug. 12, 1881:—'Considering only the earth and the moon, and overlooking minor details, we find two critical epochs in the history of the evolution of the earth and moon. At each of these epochs the day is equal to the month. The first epoch occurred, perhaps, more than 50,000,000 years ago (fifty million years). The day was then only a few hours long, at all events under six hours, and the month had the same duration. The earth and the moon were close together, and each constantly turned the same face to the other. From this initial stage the tides gradually increased both the day and the month; the latter increasing more rapidly, until a stage was reached, when the ratio of the month to the day attained a maximum. The month was then twenty-nine times the day. This maximum has recently been passed, so that while both day and month are increasing, their ratio is decreasing, and when unity is attained the final stage has been reached. The day and the month will then be each more than fifty of our present twenty-four hour days. The earth and the moon will then each present the same face constantly to the other and their distance will be much greater than at present. At both the critical epochs the earth and the moon will, in fact, move as if they were parts of one rigid body.

After the final stage has been reached, the sun's tidal action will make the day still longer than the month, so that the earth is destined to have the same wonderful relation to its satellite which Mars has to its interior satellite.'

COMPUTATION FORM.

COMPUTATION OF TIME AND HEIGHT OF HIGH WATER AT LIVERPOOL.

DATE. morning or evening.

	D.	H.	M.
Mean Time of Moon's Transit B (1½ days before date).........			
Equation of Time from Mean Time................................			
Apparent Time of Moon's Transit B, *Upper* or *Lower*			

NOTE.—If Lower, the Tide is *Lunar;* if Upper, *Anti-lunar.* If the App. Time is between 6h and 18h, the Tide is *Solar;* if between 18h and 6h, *Anti-solar;* to be marked accordingly.

		Table.	ft.	in.		Table.
For Height		VI			For Time	I.
		VIa			„	+ 12m.
Hor. Par.	' " inc. dec.	VII			„	II.
Lun. or Anti-lun. Dec.	° ' N or S. asc. or dec.	VIII			„	III.
Solar or Anti-sol. Dec.	° N or S.	IX			„	IV.
	Height required.					V.

Eqn. of T. from App. T.

Time required ....

FOR ANY OTHER PORT, to find (approximately) from these results, the time and height of High Water on any day, and also the height of the Tide at any hour of the day, consult Professor Haughton's Manual of Tides and Tidal Currents, published by Cassell Petter and Galpin, London.

THE Times and Heights of High Water are calculated from the following Tables:—

TABLE I.

Showing the Semi-monthly Inequality +, a constant; or the Interval between the Moon's Transit, two days preceding a London Tide, and the Time of High Water; the Moon's Parallax being 57′, *her Declination* 15°; *the Sun's Parallax* 8″·8, *and Declination* 15°.

☽'s Transit B.		Brest.			Portsmouth.			Dover.			Sheerness.			London.			Harwich.			☽'s Transit B.	
h	m	d	h	m	d	h	m	d	h	m	d	h	m	d	h	m	d	h	m	h	m
0	0	1	4	24	1	12	21	1	11	54	2	1	56	2	3	14	2	1	9	0	0
0	30	1	4	16	1	12	15	1	11	49	2	1	47	2	3	6	2	1	2	0	30
1	0	1	4	8	1	12	8	1	11	43	2	1	39	2	2	58	2	0	54	1	0
1	30	1	4	0	1	12	1	1	11	38	2	1	30	2	2	51	2	0	47	1	30
2	0	1	3	52	1	11	53	1	11	32	2	1	22	2	2	43	2	0	40	2	0
2	30	1	3	45	1	11	46	1	11	26	2	1	15	2	2	36	2	0	33	2	30
3	0	1	3	39	1	11	40	1	11	21	2	1	9	2	2	29	2	0	27	3	0
3	30	1	3	34	1	11	35	1	11	17	2	1	5	2	2	25	2	0	21	3	30
4	0	1	3	33	1	11	32	1	11	13	2	1	3	2	2	23	2	0	18	4	0
4	30	1	3	32	1	11	30	1	11	11	2	1	4	2	2	24	2	0	18	4	30
5	0	1	3	35	1	11	32	1	11	12	2	1	10	2	2	28	2	0	21	5	0
5	30	1	3	42	1	11	37	1	11	14	2	1	21	2	2	36	2	0	28	5	30
6	0	1	3	56	1	11	49	1	11	22	2	1	37	2	2	51	2	0	44	6	0
6	30	1	4	15	1	12	4	1	11	33	2	1	56	2	3	10	2	1	4	6	30
7	0	1	4	33	1	12	21	1	11	47	2	2	12	2	3	27	2	1	20	7	0
7	30	1	4	48	1	12	36	1	12	2	2	2	25	2	3	39	2	1	32	7	30
8	0	1	4	56	1	12	47	1	12	12	2	2	32	2	3	47	2	1	39	8	0
8	30	1	5	1	1	12	53	1	12	16	2	2	35	2	3	53	2	1	44	8	30
9	0	1	5	0	1	12	54	1	12	17	2	2	33	2	3	53	2	1	44	9	0
9	30	1	4	57	1	12	52	1	12	16	2	2	29	2	3	49	2	1	41	9	30
10	0	1	4	51	1	12	47	1	12	13	2	2	24	2	3	44	2	1	37	10	0
10	30	1	4	45	1	12	41	1	12	9	2	2	18	2	3	38	2	1	31	10	30
11	0	1	4	38	1	12	34	1	12	4	2	2	11	2	3	30	2	1	24	11	0
11	30	1	4	31	1	12	27	1	12	0	2	2	3	2	3	22	2	1	16	11	30

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Continuation of Table I.

☽'s Transit B.		Hull.			Sunderland.			North Shields.			Leith.			Thurso.			Greenock.			☽'s Transit B.	
h	m	d	h	m	d	h	m	d	h	m	d	h	m	d	h	m	d	h	m	h	m
0	0	1	19	28	1	16	18	1	16	19	1	15	15	1	9	4	1	12	48	0	0
0	30	1	19	21	1	16	10	1	16	12	1	15	8	1	8	57	1	12	42	0	30
1	0	1	19	12	1	16	2	1	16	5	1	15	0	1	8	50	1	12	34	1	0
1	30	1	19	5	1	15	55	1	15	58	1	14	53	1	8	43	1	12	26	1	30
2	0	1	18	57	1	15	48	1	15	52	1	14	47	1	8	37	1	12	18	2	0
2	30	1	18	51	1	15	42	1	15	46	1	14	40	1	8	31	1	12	11	2	30
3	0	1	18	45	1	15	38	1	15	41	1	14	36	1	8	28	1	12	4	3	0
3	30	1	18	40	1	15	36	1	15	37	1	14	34	1	8	26	1	11	58	3	30
4	0	1	18	39	1	15	35	1	15	38	1	14	34	1	8	26	1	11	55	4	0
4	30	1	18	44	1	15	37	1	15	42	1	14	36	1	8	28	1	11	54	4	30
5	0	1	18	54	1	15	42	1	15	50	1	14	45	1	8	35	1	11	56	5	0
5	30	1	19	6	1	15	52	1	16	3	1	14	56	1	8	47	1	12	2	5	30
6	0	1	19	18	1	16	8	1	16	20	1	15	13	1	9	6	1	12	15	6	0
6	30	1	19	31	1	16	23	1	16	35	1	15	30	1	9	28	1	12	30	6	30
7	0	1	19	42	1	16	36	1	16	49	1	15	42	1	9	43	1	12	46	7	0
7	30	1	19	53	1	16	45	1	16	59	1	15	52	1	9	54	1	13	0	7	30
8	0	1	20	1	1	16	51	1	17	3	1	15	58	1	9	59	1	13	9	8	0
8	30	1	20	4	1	16	54	1	17	4	1	15	58	1	9	59	1	13	14	8	30
9	0	1	20	2	1	16	55	1	17	1	1	15	55	1	9	54	1	13	15	9	0
9	30	1	19	58	1	16	53	1	16	57	1	15	52	1	9	47	1	13	13	9	30
10	0	1	19	53	1	16	49	1	16	50	1	15	47	1	9	38	1	13	10	10	0
10	30	1	19	48	1	16	42	1	16	42	1	15	40	1	9	29	1	13	6	10	30
11	0	1	19	42	1	16	34	1	16	34	1	15	33	1	9	21	1	13	1	11	0
11	30	1	19	35	1	16	26	1	16	27	1	15	24	1	9	12	1	12	55	11	30

☽'s Transit B.		Liverpool.			Pembroke.			Weston-super-mare.			Holyhead.			Kingstown.			Belfast.			☽'s Transit B.	
	m	d	h	m	d	h	m	d	h	m	d	h	m	d	h	m	d	h	m	h	m
0	0	1	12	0	1	6	51	1	7	34	1	10	47	1	11	47	1	11	25	0	0
0	30	1	11	52	1	6	43	1	7	26	1	10	38	1	11	41	1	11	16	0	30
1	0	1	11	45	1	6	35	1	7	18	1	10	30	1	11	34	1	11	9	1	0
1	30	1	11	37	1	6	27	1	7	9	1	10	24	1	11	27	1	11	3	1	30
2	0	1	11	29	1	6	20	1	7	0	1	10	18	1	11	21	1	10	57	2	0
2	30	1	11	21	1	6	13	1	6	51	1	10	14	1	11	15	1	10	52	2	30
3	0	1	11	15	1	6	7	1	6	42	1	10	10	1	11	10	1	10	49	3	0
3	30	1	11	9	1	6	0	1	6	33	1	10	7	1	11	7	1	10	48	3	30
4	0	1	11	6	1	5	55	1	6	24	1	10	7	1	11	6	1	10	49	4	0
4	30	1	11	5	1	5	51	1	6	18	1	10	9	1	11	7	1	10	51	4	30
5	0	1	11	10	1	5	50	1	6	17	1	10	16	1	11	15	1	10	57	5	0
5	30	1	11	20	1	5	50	1	6	22	1	10	28	1	11	26	1	11	7	5	30
6	0	1	11	37	1	5	58	1	6	34	1	10	46	1	11	38	1	11	20	6	0
6	30	1	11	59	1	6	11	1	6	50	1	11	3	1	11	51	1	11	33	6	30
7	0	1	12	16	1	6	32	1	7	7	1	11	16	1	12	3	1	11	48	7	0
7	30	1	12	28	1	6	51	1	7	23	1	11	26	1	12	13	1	12	0	7	30
8	0	1	12	35	1	7	3	1	7	37	1	11	31	1	12	21	1	12	6	8	0
8	30	1	12	37	1	7	10	1	7	48	1	11	33	1	12	26	1	12	7	8	30
9	0	1	12	35	1	7	14	1	7	54	1	11	31	1	12	28	1	12	5	9	0
9	30	1	12	31	1	7	15	1	7	56	1	11	26	1	12	26	1	12	1	9	30
10	0	1	12	25	1	7	14	1	7	55	1	11	20	1	12	19	1	11	56	10	0
10	30	1	12	20	1	7	11	1	7	52	1	11	13	1	12	11	1	11	49	10	30
11	0	1	12	13	1	7	5	1	7	47	1	11	6	1	12	3	1	11	42	11	0
11	30	1	12	6	1	6	57	1	7	41	1	10	57	1	11	55	1	11	34	11	30

Continuation of Table I.

☽'s Transit B.	Londonderry.	Sligo.	Galway.	Queenstown.	Waterford.	☽'s Transit B.
h m	d h m	d h m	d h m	d h m	d h m	h m
0 0	1 8 38	1 5 55	1 5 14	1 5 41	1 6 1	0 0
0 30	1 8 28	1 5 46	1 5 7	1 5 34	1 5 54	0 30
1 0	1 8 19	1 5 39	1 5 0	1 5 26	1 5 47	1 0
1 30	1 8 10	1 5 32	1 4 53	1 5 19	1 5 40	1 30
2 0	1 8 1	1 5 25	1 4 46	1 5 11	1 5 32	2 0
2 30	1 7 54	1 5 18	1 4 40	1 5 4	1 5 24	2 30
3 0	1 7 48	1 5 11	1 4 35	1 4 57	1 5 16	3 0
3 30	1 7 48	1 5 8	1 4 32	1 4 50	1 5 9	3 30
4 0	1 7 54	1 5 9	1 4 31	1 4 45	1 5 2	4 0
4 30	1 8 5	1 5 15	1 4 32	1 4 43	1 4 57	4 30
5 0	1 8 20	1 5 25	1 4 36	1 4 42	1 4 59	5 0
5 30	1 8 40	1 5 38	1 4 46	1 4 46	1 5 9	5 30
6 0	1 8 57	1 5 53	1 5 1	1 5 0	1 5 20	6 0
6 30	1 9 10	1 6 7	1 5 19	1 5 16	1 5 32	6 30
7 0	1 9 19	1 6 21	1 5 33	1 5 32	1 5 44	7 0
7 30	1 9 22	1 6 32	1 5 44	1 5 47	1 5 56	7 30
8 0	1 9 21	1 6 38	1 5 49	1 5 59	1 6 8	8 0
8 30	1 9 18	1 6 38	1 5 50	1 6 6	1 6 18	8 30
9 0	1 9 15	1 6 34	1 5 50	1 6 8	1 6 23	0 0
9 30	1 9 12	1 6 28	1 5 48	1 6 7	1 6 25	9 30
10 0	1 9 8	1 6 22	1 5 42	1 6 4	1 6 24	10 0
10 30	1 9 3	1 6 16	1 5 36	1 5 59	1 6 22	10 30
11 0	1 8 56	1 6 10	1 5 29	1 5 53	1 6 16	11 0
11 30	1 8 4[illegible]	1 6 3	1 5 21	1 5 47	1 6 8	11 30

TABLE II.

Showing the Correction for the Moon's Parallax.

☽'s Transit B.	H. P. 54′	H. P. 55′	H. P. 56′	H. P. 57′	H. P. 58′	H. P. 59′	H. P. 60′	H. P. 61′	☽'s Transit B.
h	m	m	m	m	m	m	m	m	h
0	+1	+1	+1	0	0	−1	−1	−1	0
1	−1	−1	−1	0	0	+1	+1	+1	1
2	−3	−2	−1	0	+1	+2	+3	+4	2
3	−5	−3	−1	0	+1	+3	+5	+7	3
4	−7	−5	−2	0	+2	+4	+6	+8	4
5	−9	−6	−3	0	+2	+5	+7	+9	5
6	−4	−2	−1	0	+1	+2	+3	+4	6
7	+4	+2	+1	0	−1	−2	−3	−4	7
8	+9	+6	+3	0	−2	−5	−7	−9	8
9	+7	+5	+2	0	−2	−4	−6	−8	9
10	+5	+3	+1	0	−1	−3	−5	−7	10
11	+3	+2	+1	0	−1	−2	−3	−4	11

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TABLE III.

Showing the Correction for the Moon's Declination.

☽'s Transit B.	0° Dec.	3° Dec.	6° Dec.	9° Dec.	12° Dec.	15° Dec.	18° Dec.	21° Dec.	24° Dec.	27° Dec.	30° Dec.	☽'s Transit B.
h	m	m	m		m	m	m	m	m	m	m	h
0	−1	−1	0	0	0	0	+1	+1	+1	+1	+ 2	0
1	+1	+1	0	0	0	0	−1	−1	−1	−1	− 2	1
2	+2	+2	+1	+1	+1	0	−1	−1	−2	−3	− 4	2
3	+3	+3	+3	+2	+1	0	−1	−2	−3	−5	− 7	3
4	+3	+3	+3	+2	+1	0	−1	−3	−5	−7	−10	4
5	+3	+3	+3	+2	+1	0	−2	−4	−6	−9	−12	5
6	+2	+2	+2	+1	+1	0	−1	−2	−4	−4	− 5	6
7	−2	−2	−2	−1	−1	0	+1	+2	+4	+4	+ 5	7
8	−3	−3	−3	−2	−1	0	+2	+4	+6	+9	+12	8
9	−3	−3	−3	−2	−1	0	+1	+3	+5	+7	+10	9
10	−3	−3	−3	−2	−1	0	+1	+2	+3	+5	+ 7	10
11	−2	−2	−1	−1	−1	0	+1	+1	+2	+3	+ 4	11

TABLE IV.

Showing the Correction for the Sun's Declination.

☽'s Transit B.	0° Dec.	3° Dec.	6° Dec.	9° Dec.	12° Dec.	15° Dec.	18° Dec.	21° Dec.	24° Dec.	☽'s Transit B.
h	m	m	m	m	m	m	m	m	m	h
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	+1	1
2	−1	−1	−1	−1	0	0	0	+1	+2	2
3	−2	−1	−1	−1	0	0	+1	+2	+4	3
4	−3	−2	−1	−2	0	0	+2	+3	+5	4
5	−3	−2	−2	−1	−1	0	+2	+3	+5	5
6	−1	−1	−1	−1	0	0	+1	+2	+2	6
7	+1	+1	+1	+1	0	0	−1	−2	−2	7
8	+3	+2	+2	+1	+1	0	−2	−3	−5	8
9	+3	+2	+1	+2	0	0	−2	−3	−5	9
10	+2	+1	+1	+1	0	0	−1	−2	−4	10
11	+1	+1	+1	+1	0	0	0	−1	−2	11

TABLE V.

Showing the Correction for the Sun's Parallax.

☽'s Transit B.	Jan. Dec.	Feb. Nov.	March Oct.	April Sept.	May August	June July	☽'s Transit B.
	H. P. 8·94″	H. P. 8·90″	H. P. 8·84″	H. P. 8·76″	H. P. 8·70″	H. P. 8·66″	
h	m	m	m	m	m	m	h
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	1
2	−1	−1	0	0	0	0	2
3	−1	−1	0	0	+1	+1	3
4	−2	−2	0	0	+2	+2	4
5	−3	−2	−1	+1	+3	+3	5
6	−2	−1	−1	+1	+2	+2	6
7	+2	+1	+1	−1	−2	−2	7
8	+3	+2	+1	−1	−3	−3	8
9	+2	+2	0	0	−2	−2	9
10	+1	+1	0	0	−1	−1	10
11	+1	+1	0	0	0	0	11

TABLE VI.

Showing the Semi-monthly Inequality +, a constant, in the Height of High Water, with reference to the apparent time of the Moon's Transit B, the Moon's Parallax being 57′, and her Declination 15°; the Sun's Parallax 8″·8, and Declination 15°.

☽'s Transit B.	Brest.	Ports-mouth.	Dover.	Sheerness.	London.	Harwich.	☽'s Transit B.
h m	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	h m
0 0	19·06	13·50	18·66	16·10	20·71	11·56	0 0
0 30	19·06	13·40	18·68	16·05	20·65	11·48	0 30
1 0	18·92	13·30	18·61	15·96	20·57	11·38	1 0
1 30	18·66	13·15	18·45	15·83	20·47	11·25	1 30
2 0	18·36	13·00	18·24	15·59	20·25	11·10	2 0
2 30	17·93	12·84	17·97	15·32	19·95	10·92	2 30
3 0	17·31	12·55	17·54	14·98	19·52	10·72	3 0
3 30	16·62	12·21	17·03	14·56	19·01	10·50	3 30
4 0	15·88	11·88	16·48	14·17	18·48	10·28	4 0
4 30	15·13	11·55	15·93	13·81	17·93	10·07	4 30
5 0	14·47	11·17	15·36	13·46	17·54	9·88	5 0
5 30	14·02	10·92	14·82	13·14	17·31	9·75	5 30
6 0	13·83	10·71	14·51	13·10	17·33	9·72	6 0
6 30	13·84	10·63	14·43	13·18	17·44	9·79	6 30
7 0	14·08	10·80	14·72	13·48	17·63	9·95	7 0
7 30	14·55	11·13	15·20	13·84	17·90	10·17	7 30
8 0	15·17	11·50	15·75	14·21	18·38	10·43	8 0
8 30	15·88	11·84	16·29	14·61	18·90	10·70	8 30
9 0	16·66	12·30	16·85	14·96	19·41	10·94	9 0
9 30	17·37	12·63	17·34	15·32	19·75	11·18	9 30
10 0	18·03	12·88	17·78	15·61	20·02	11·38	10 0
10 30	18·56	13·13	18·17	15·84	20·26	11·51	10 30
11 0	18·84	13·30	18·41	16·03	20·50	11·57	11 0
11 30	19·01	13·42	18·58	16·13	20·73	11·59	11 30

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Continuation of Table VI.

☽'s Transit B.	Hull.	Sunderland.	North Shields.	Leith.	Thurso.	Greenock.	☽'s Transit B.
h m	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	h m
0 0	20·87	14·43	13·25	16·29	13·25	9·72	0 0
0 30	20·84	14·41	13·13	16·18	13·17	9·74	0 30
1 3	20·75	14·26	12·95	16· 0	13· 0	9·71	1 0
1 30	20·54	14·02	12·75	15·78	12·77	9·68	1 30
2 0	20·17	13·71	12·53	15·54	12·46	9·61	2 0
2 30	19·70	13·38	12·29	15·25	12·09	9·48	2 30
3 0	19·18	13·01	11·95	14·88	11·67	9·33	3 0
3 30	18·62	12·64	11·59	14·42	11·23	9·17	3 30
4 0	18·05	12·22	11·14	13·92	10·76	9· 0	4 0
4 30	17·49	11·82	10·64	13·46	10·35	8·13	4 30
5 0	16·96	11·46	16·25	13·05	9·98	8·64	5 0
5 30	16·52	11·18	10·00	12·73	9·67	8·44	5 30
6 0	16·34	11·02	9·96	12·58	9·52	8·29	6 0
6 30	16·39	11·03	10·06	12·67	9·50	8·19	6 30
7 0	16·78	11·25	10·31	12·87	9·61	8·29	7 0
7 30	17·43	11·64	10·62	13·21	9·84	8·46	7 30
8 0	18·06	12·09	10·96	13·60	10·30	8·67	8 0
8 30	18·66	12·54	11·33	14·08	10·84	8·85	8 30
9 0	19·22	12·95	11·72	14·61	11·45	9·04	9 0
9 30	19·70	13·31	12·17	15·14	12· 0	9·18	9 30
10 0	20·09	13·64	12·53	15·59	12·52	9·31	10 0
10 30	20·39	13·91	12·83	15·92	12·93	9·44	10 30
11 0	20·63	14·14	13·06	16·18	13·18	9·56	11 0
11 30	20·80	14·33	13·19	16·30	13·25	9·66	11 30

☽'s Transit B.	Liverpool.*	Pembroke.	Weston-super-mare.	Holyhead.	Kingstown.	Belfast.	☽'s Transit B.
h m	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	h m
0 0	26·21	21. 0	37·27	16· 0	10·94	9·43	0 0
0 30	26·21	21· 0	37·25	15·94	10·88	9·40	0 30
1 0	26· 0	20·82	37·09	15·83	10·80	9·37	1 0
1 30	25·55	20·58	36·82	15·66	10·68	9·32	1 30
2 0	25·05	20·24	36·37	15·41	10·49	9·24	2 0
2 30	24·46	19·81	35·75	15·09	10·28	9·14	2 30
3 0	23·80	19·25	34·83	14·68	10·05	9· 0	3 0
3 30	23·13	18·60	33·80	14·26	9·84	8·83	3 30
4 0	22·38	17·96	32·67	13·84	9·61	8·63	4 0
4 30	21·63	17·33	31·51	13·41	9·38	8·45	4 30
5 0	20·88	16·58	30·34	13·03	9·13	8·30	5 0
5 30	20·30	15·92	29·40	12·73	8·92	8·18	5 30
6 0	20· 0	15·63	28·79	12·63	8·83	8·09	6 0
6 30	20·21	15·56	28·75	12·73	8·91	8·06	6 30
7 0	20·63	15·77	29·08	12·97	9·10	8·07	7 0
7 30	21·21	16·30	29·83	13·27	9·32	8·17	7 30
8 0	22· 0	17· 0	30·79	13·66	9·55	8·36	8 0
8 30	22·71	17·81	31·91	14·11	9·79	8·61	8 30
9 0	23·63	18·54	33·16	14·55	10·04	8·86	9 0
9 30	24·30	19·23	34·29	14·95	10·28	9·06	9 30
10 0	24·80	19·81	35·25	15·30	10·50	9·22	10 0
10 30	25·30	20·27	36· 0	15·60	10·70	9·33	10 30
11 0	25·63	20·62	36·50	15·80	10·86	9·41	11 0
11 30	26· 0	20·89	36·95	15·93	10·96	9·44	11 30

* + a constant, see Table VI*a*., page xx.

TABLE Ia.

Shewing the Semi-monthly Inequality in the Time of High Water at Liverpool, when Transit (A) is employed.

☽'s Transit A.	Interval.	☽'s Transit A.	Interval.	☽'s Transit A.	Interval.	☽'s Transit A.	Interval.
h m		h m		h m		h m	
0 0	2 0 32·6	3 0	1 23 52·8	6 0	2 0 32·4	9 0	2 1 13·7
0 30	2 0 24·5	3 30	1 23 49·2	6 30	2 0 50·2	9 30	2 1 9·8
1 0	2 0 17·0	4 0	1 23 50·7	7 0	2 1 1·9	10 0	2 1 3·0
1 30	2 0 9·5	4 30	1 23 52·2	7 30	2 1 13·7	10 30	2 0 56·1
2 0	2 0 2·9	5 0	2 0 3·4	8 0	2 1 15·6	11 0	2 0 48·4
2 30	1 23 56·3	5 30	2 0 14·7	8 30	2 1 17·6	11 30	2 0 40·8

EXAMPLES OF THE USE OF THE TABLES.

		d. h. m.
Apparent time of Moon's Transit (B) April		- 5 1 15
Table I. gives -	-	- 1 11 53
II. „	−1	
III. „	0	
IV. „	0	- + 2
V. „	0	
Equation of Time	+3	
		6 13 10

which gives 1h. 10m. a.m. for the Greenwich Mean Time of High Water at Liverpool on the morning of April 6, 1882.

NOTE.—12 minutes are added to Table I. to obtain Greenwich Time. In Table I*a*. they are already included.

The factors by which the heights computed for Liverpool must be multiplied in order to obtain the heights for other ports are approximately the ratios of the Mean. Spring Ranges, and as are as follows:—

For Brest -	$\frac{38}{55}$	For North Shields -	$\frac{27}{55}$	For Belfast -	$\frac{19}{55}$
Devonport -	$\frac{31}{55}$	Leith -	$\frac{3}{5}$	Londonderry -	$\frac{3}{11}$
Portsmouth -	$\frac{27}{55}$	Thurso -	$\frac{26}{55}$	Sligo -	$\frac{2}{5}$
Dover -	$\frac{37}{55}$	Greenock -	$\frac{19}{55}$	Galway -	$\frac{6}{11}$
Sheerness -	$\frac{32}{55}$	Pembroke -	$\frac{42}{55}$	Queenstown -	$\frac{23}{55}$
Harwich -	$\frac{23}{55}$	Weston-sup-Mare	$1\frac{19}{55}$	Waterford -	$\frac{24}{55}$.
Hull - -	$\frac{42}{55}$	Holyhead -	$\frac{32}{55}$		
Sunderland -	$\frac{29}{55}$	Kingstown -	$\frac{2}{5}$		

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The following abridged tables for computing the heights of high water at Liverpool, based on Sir J. W. Lubbock's calculations, are by the Rev. J. Pearson, M.A., F.R.A.S., of Fleetwood, late Scholar of Trinity College, Cambridge, from 10 years' observations.

TABLE VI*a*.

Showing the correction for Lunar or Anti-lunar tide.

☽'s Transit B.	January.		February.		March.		April.		May.		June.	
	Lunar.	Anti-lunar.	Lunar.	Anti-lunar.	Lunar.	Anti-lunar.	Lunar.	Anti-lunar.	Lunar.	Anti-lunar.	Lunar.	Anti-lunar.
h						Inches.						
0	+5	0	+5	+1	+6	+2	+6	+2	+4	+1	+3	0
3	+5	+1	+5	+1	+4	+1	+4	+1	+4	+2	+5	+1
6	+6	+3	+4	0	+2	−2	+4	−1	+4	0	+5	+2
9	+5	+3	+5	+2	+5	+1	+5	+1	+3	+1	+3	−1
12	+5	+2	+6	+2	+6	+3	+4	+1	+3	0	+2	−1
15	+6	+3	+6	+2	+5	+1	+3	+0	+3	0	+3	−1
18	+5	+2	+4	0	+2	−4	+3	−2	+4	+1	+5	+1
21	+4	+0	+5	0	+5	+1	+5	+1	+5	+1	+5	+2
	July.		August.		September.		October.		November.		December.	
0	+4	+1	+5	+2	+7	+3	+6	+2	+3	+1	+3	+1
3	+5	+1	+4	+2	+4	+1	+5	+1	+4	0	+3	+1
6	+5	+2	+3	−1	+3	−2	+3	0	+5	0	+5	+2
9	+3	−1	+3	−1	+4	0	+4	0	+5	+1	+6	+3
12	+3	−2	+4	0	+5	+1	+5	+2	+5	+1	+5	+1
15	+4	−1	+4	0	+5	+1	+5	+2	+6	+2	+5	+2
18	+5	+1	+4	+1	+3	0	+4	+1	+5	+1	+7	+3
21	+4	0	+5	+1	+5	+2	+5	+1	+4	+1	+4	+2

TABLE VII.

Showing the Correction for the Moon's Parallax.

☽'s Transit B.	Increasing.					Decreasing.			
	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
	53′	55′	57′	59′	61′	59′	57′	55′	53′
h h					Inches.				
11 to 14 23 2	−18	− 6	+ 4	+16	+25	+18	+ 5	− 4	−16
2 5 7 11 14 17 19 23	−19	− 7	+ 3	+15	+24	+17	+ 4	− 5	−17
5 7 17 19	−20	− 8	+ 2	+14	+23	+16	+ 3	− 6	−18

TABLE VIII.

Showing the correction for Lunar or Anti-lunar declination.

Computed for the mean parallax of 57′, and for a mean inclination of orbit to the equator of 24°.*

North Declination (Ascending).								
0	3°	6°	9°	12°	15°	18°	21°	24°
				Inches.				
+13	+12	+11	+8	+5	+2	−1	−6	−12
North Declination (Descending).								
24°	21°	18°	15°	12°	9°	6°	3°	0
−12	−8	−5	−4	−2	0	+2	+3	+4
South Declination (Descending).								
0	3°	6°	9°	12°	15°	18°	21°	24°
+4	+3	+2	0	−1	−2	−3	−6	−7
South Declination (Ascending).								
24°	21°	18°	15°	12°	9°	6°	3°	0
−7	−2	+2	+6	+8	+10	+12	+13	+14

* With the parallax ranging from 53′ to 57′, the increments must be decreased one inch; with the parallax ranging from 57′ to 61′, the increments must be increased one inch. Alterations due to change in the inclination of orbit require separate tables.

TABLE IX.*

Showing the correction for Solar or Anti-solar Declination.

***Solar** or Anti-solar declination, South.* >< *Solar or Anti-Solar declination, North.* >< *Solar or Anti-solar declination, South.*

Dec.	☽'s Transit B. Hours.								
	0	3	6	9	12	15	18	21	24
	12	15	18	21	0	3	6	9	12
	Inches.								
24°	+3	+4	+5	+2	+1	+2	+5	+5	+3
21°	+4	+4	+5	+2	+2	+2	+5	+5	+4
18°	+4	+5	+5	+2	+2	+2	+4	+5	+4
15°	+5	+5	+4	+2	+3	+2	+3	+4	+5
12°	+5	+4	+3	+3	+4	+3	+2	+4	+5
9°	+6	+4	+2	+3	+4	+3	+1	+4	+6
6°	+6	+4	0	+3	+5	+3	+1	+4	+6
3°	+6	+5	−1	+3	+5	+4	0	+4	+6
0°	+7	+5	−2	+4	+6	+4	−2	+5	+7

* Computed for the summer months; for the winter months, the corrections must be slightly increased.

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The foregoing tables for Liverpool are deduced from the action of four separate tides, super-imposed in pairs, and which are:—

1. A lunar tide caused by the direct action of the moon.
2. An anti-lunar tide caused by the obverse action of the moon.
3. A solar tide due to the direct action of the sun.
4. An anti-solar tide due to the obverse action of the sun.

For transit B, *see* page xv.

Lower transits (B) are followed by lunar tides; upper transits (B) by anti-lunar tides.

All transits (B) occurring between 0 h. and 12 h. (apparent Greenwich time) give morning tides; those between 12 h. and 24 h., afternoon tides; those between 6 h. and 18 h. are connected with solar tides; and those between 18 h. and 6 h. with anti-solar tides.

For anti-lunar tides the declination must be reversed as regards north and south, ascending and descending, from that governing lunar tides; that is, from the declination given in the Nautical Almanac.

In like manner, for anti-solar tides, the declination as regards north and south must also be reversed.

Example of the Use of the Rev. J. Pearson's Tables.

Apparent time of the Moon's Lower Transit (B), April 5th
(*Lunar and Anti-solar*) - - - 1 h. 15 m.

			ft.	in.
	Table VI. gives - - -		25	10
H.P. 55′ 32′ inc.	„ VI*a*. „ - - -	+ 5	+	6
Lunar Dec. 15° 36′ S., des^{g}	„ VII. „ - - -	— 3		
Anti-solar Dec. 6° S.	„ VIII. „ - - -	— 2		
	„ IX. „ - - -	+ 6		
			26	4

26 feet 4 inches being the calculated height of high water at Liverpool on 7th April, A.M., above the Admiralty datum.

When the Old Dock Sill datum is used, as at George Pier, subtract 8 feet 6 inches; at Helbre Island, subtract 9 feet 6 inches.

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